A Model of Catch-up Growth Through Productivity Diffusion*

Michael Kirker†

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Abstract

The diffusion of productivity ideas and technology from richer, more R&D intensive countries is an important source of growth for many small open economies. I develop a two-country, two-sector model that incorporates this channel for the diffusion of productivity ideas into a less developed small country. A calibrated version of this model is able to replicate the broad pattern of catch-up growth observed in the data for open economies when around one percent of the learning opportunities in the home country comes from an overseas source. Despite this low meeting rate, the diffusion of productivity ideas drives nearly all of the increase in average productivity in those countries furthest from the productivity frontiers during the first year, but drives little of the increase in productivity within countries close to the productivity frontier. I also consider how tariffs may interfere with the diffusion of productivity through a selection effect, limiting the ability of productive foreign firms to operate in the home country. Despite this effect, I find that the introduction of a 30 percent tariff is optimal for a small under-developed country in terms of maximizing GDP.

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†Email: mkirker@uchicago.edu

1 Introduction

Openness to trade is an important factor for growth in many countries. Not only does trade bring more demand for goods currently being produced in the country, but it also allows an inflow of new ideas and technology from the rest of the world. Eaton and Kortum (2001) show that both global R&D and production of capital goods is highly concentrated in a small number of developed countries such as Germany, Japan, USA, Great Britain, and Sweden. By trading with highly intensive R&D countries, smaller open economies can also benefit from the advancements in production technology that are discovered. The influence of foreign innovations can be a large driver of growth in smaller countries. According to estimates by Eaton and Kortum (1996), every OECD country besides the United States receives over 50 percent of its productivity growth from overseas sources.

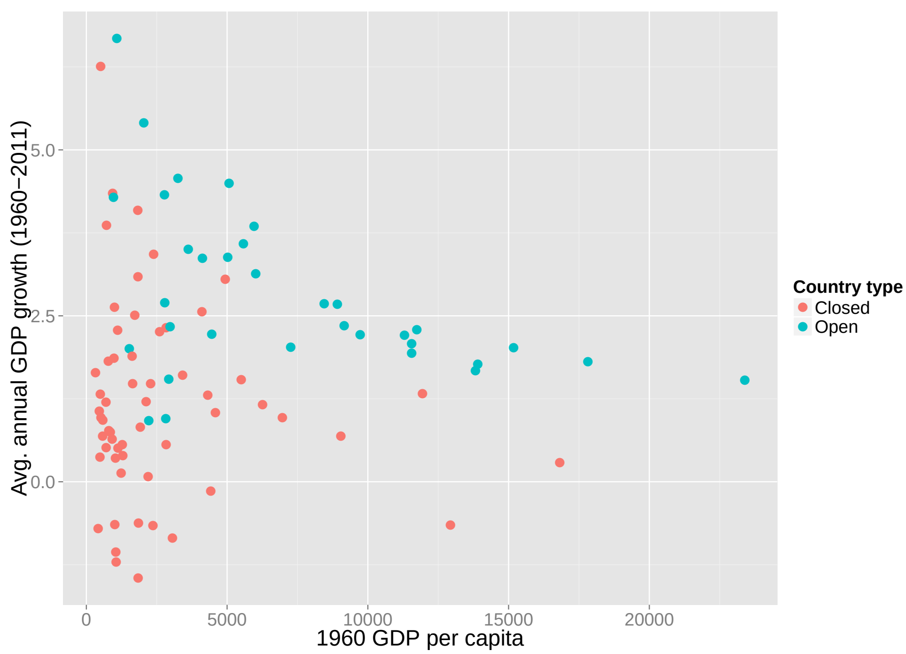
Figure 1 reproduces one of the figures from Lucas (2009b) using the most recent data from the Penn World Tables 8.0. The figure shows the average annual growth rate in GDP per capita between 1960 and 2011 against the 1960 level of GDP per capita for a panel of countries. The countries in the panel are divided into two groups, open and closed, based on the classification of Sachs and Warner (1995) who use a relatively restrictive definition of open.[1]

Two key features stand out in figure 1. First, countries who were relatively poor in 1960 show a much larger variation in growth rates relative to richer countries. The poorest countries have growth rates spanning from -1.1% to 6.7%, while the richest countries tend to have less variation in their growth rates, with an average growth rate close to two percent for this richest countries. This creates a triangular shaped distribution of growth within the sample. Second, those countries that are open have tended to grow faster since 1960 than closed countries. Most open countries lie on the upper edge of the triangular shape, forming an asymptoting curve. This curve implies that for most open countries income levels have converged over time. Growth regression analysis developed by Barrow (1991) and Mankow et al. (1992) also conclude that catch-up growth has occurred across countries.

[1] According to the criteria of Sachs and Warner (1995), a country is classified as closed if any of the following five conditions are met: (1) its average tariff rate exceeded 40%, (2) its non-tariff barriers covered more than 40% of imports, (3) it had a socialist economic system (4) it had a state monopoly of major exports, or (5) its black-market premium exceeded 20% during either the 1970s or 1980s.

The classification of open and closed economies by Sachs and Warner (1995) is similar to the classification if one considers a more graduated definition such as the trade-to-GDP ratio, or the International Chamber of Commerce's measure of openness.

Figure 1: Income and Growth Rates in 68 countries



Each point refers to an individual country. Countries are classified as open by Sachs and Warner (1995) tend to lie on the upper edge of the triangular distribution of growth rates, with poorer open countries growing faster that richer open countries, implying convergence in income levels for open countries.

All variables are expressed in P.P.P. terms.

Lucas (2009b) develops a single differential equation to fit the growth rates of the open countries that lie on the upper edge of the distribution in figure 1. The differential equation is based on the idea of a constant spillover of human capital knowledge from a leading economy to a poorer country. This model assumes GDP is proportional to human capital and lacks the type of economic structure commonly used in international macroeconomic models. More recently, Alvarez et al. (2014) illustrates how to incorporate a model of productivity diffusion among countries into an Eaton and Kortum (2002) style trade model. However, the growth rate in the model is too responsive to the opening up of trade relative to the data.

In this paper, I develop an economic model of productivity diffusion that is able to match the pattern of catch-up growth seen for most of the open economy countries in figure 1. The model extends the productivity diffusion process in papers such as Kortum (1997) and Alvarez et al. (2014) to apply to the two-country, two-sector general equilibrium modeling framework used in many international macroeconomic papers. Heterogeneous firms in home country improve their productivity over time by applying new production techniques learned from both other firms within the home country and from overseas firms who export to the home country. By being open to trade, the home country is able to converge to the productivity frontier over time, leading to the convergence in GDP per capita.

A calibrated version of the model is able to match the growth rates of the open economy countries along the upper edge of the distribution in figure 1 when around one percent of the opportunities that a firm in the home country has to improve its own productivity comes from learning from overseas sources. Despite the low rate of meetings between the home country and foreign country firms required by the model to match the catch-up growth profile in the data, I show that the foreign country still has a significant influence on the growth in average productivity in poorer home countries in the first period after opening up to trade. In the case of the poorest country considered, around 80 percent of the growth in average tradable productivity, and around 99 percent of the growth in average non-tradable productivity is driven from the diffusion of productivity ideas from overseas.

In this paper I also consider how import tariffs create a dynamic trade-off for small countries in terms of the intra-temporal gains and the inter-temporal losses. Increasing tariffs on imports helps protect the home country's tradable firms against competition from more productive foreign firms. This benefits the home country by shifting the demand of home country households away from imports, and towards locally produced tradable goods. However, by replacing productive foreign firms with less productive local firms, the distribution of productivity of firms operating in the home country is made worse. As a result, the probability of home country firms to meet and learn from a more productive firm operating in the home country is reduced, slowing the growth rate of the home country. Ex-ante, it is not clear which of these two forces will dominate.

Simulations from an extension of the model that includes tariffs reveals that intra-temporal gains outweigh the inter-temporal losses for a wide range of tariffs (tariff rates below 90 percent) for a country that is one percent of the world's population, and starts with a level of GDP per capita of only 10 percent of the rest of the world. The optimal tariff rate for such a country is around 30 percent, but the gains in growth only result in a modest eight percent improvement in the log of GDP per capita after the 51 years of simulation (1960 to 2011). Given the relatively small size of the home country, the gains in domestic demand from the tariff are not significantly large. And in addition, with the large initial productivity gap, most foreign firms are able to overcome even sizable tariffs, which limits ability of the tariff to significantly affect the productivity distribution of firms operating in the home country.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 details the static trade model that maps the distribution of productivity within a country into macroeconomic variables. Section 4 describes the diffusion process by which the distributions of productivity evolve over time within the model. Section 5 discusses how the model is calibrated to the data. Section 6 discusses the simulation results from the calibrated model. Section 7 extends the model to analyze optimal tariff policy. And section 8 concludes.

2 Literature Review

This paper builds upon the canonical two-country, two-sector general-equilibrium model that underlies the International Real Business Cycle (IRBC) and New Open Economy Macroeconomics (NOEM) models, and has been well documented in the literature. The extensions to the model draw on two separate areas of the literature. The first being Ricardian trade models, and the second being processes of productivity diffusion. Below I discuss those papers from these two areas of the literature that are most closely related to the approach taken in this paper.

The model used in this paper shares features in common with recent static Ricardian models such as those used by Eaton and Kortum (2002) and Alvarez and Lucas (2007). Like the models in these two previous papers, the model employed in this paper features iceberg trade costs as well as firms that are heterogeneous in their productivity levels. Iceberg trade costs assist in creating home-bias in consumption as well as allowing for the possibility that the home country can converge to a level of income that is different to that in the foreign country on the balanced growth path (when trade costs are asymmetrical). Including firms that are heterogeneous in terms of productivity levels in each sector is vital to incorporate the chosen productivity diffusion process within the models structure. However, the models developed by Eaton and Kortum (2002) and Alvarez and Lucas (2007) are used to focus on the gains from trade that arise from eliminating trade costs in a static multi-country framework. I instead use the model framework to focus on the inter-temporal dimension.

The mathematical process of productivity diffusion and growth used in this paper is similar to an earlier paper by Kortum (1997) in which agents learn from an external distribution of productivity ideas. It is also based on the continuous-time process described by Alvarez et al. (2008) who build upon earlier work by Eaton and Kortum (1999) to analyze the long run implications of endogenous and exogenous idea growth in both the discrete and continuous time cases. Their analysis of the continuous time case forms the foundation for the productivity diffusion process in this paper. However, I extend the productivity diffusion process of Alvarez et al. (2008) to include the case where there is contemporaneous learning from both internal and external sources, and the source of external productivity ideas is endogenously growing over time.

The paper by Alvarez et al. (2014) lies closest to the approach taken in this paper. Alvarez et al. (2014) incorporate the diffusion of productivity ideas discussed above into the Ricardian framework of Eaton and Kortum (2002) and Alvarez and Lucas (2007). Using this model, Alvarez et al. (2014) examine the dynamics of trade costs in both the short and long run for an n-country world. However, the numerical analysis in this paper serves more to illustrate the channels of the model than to provide an estimated description of the world. The response of GDP to an opening up of trade is too rapid to be able to match data. The model that I develop in this paper is intended to be able to be taken to the data without significant introduction of frictions in the learning process, to see if this class of models is able to match the data for reasonable parameterizations.

3 Static Trade Model

The economy model developed in this paper consists of two parts. The first part, discussed in this section, is a static trade model that provides a mapping from the country's productivity distributions into macroeconomic variables that exist in the data. The second part, discussed in the next section, is a diffusion model that provides a description for how the distribution of productivity in a country evolves over time.

The static trade model reflects the intra-temporal dynamics of a static two-country, two-sector world. The home country is assumed to be a small open economy (SOE) that is a price taker in global markets. The foreign country, which represents the rest of the world as well as the production frontier, is assumed to be sufficiently large that it is unaffected by developments in the home country, and can thus be represented as a closed economy.[2] The main advantage of using the small open economy framework for this model is that the the we can interpret the dynamics of the foreign economy as exogenous when viewed from the point of view of the home country. Therefore, it can be thought of as the leading economy that is on the Balance Growth Path (BGP) and productivity frontier in all periods.

3.1 Home Country

The home country is populated by a continuum of consumers of measure one. Each home-country consumer has identical preferences over composite (bundled) non-tradable (C_{NT}) and tradable (C_{TR}) goods given by the constant elasticity of substitution (CES) utility function

$$C = \left[\omega^{1/\eta} C_{NT}^{(\eta-1)/\eta} + (1-\omega)^{1/\eta} C_{TR}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)} \tag{1}$$

where η is the elasticity of substitution between tradable and non-tradable composite goods in the household's consumption, and ω is a measure of degree of non-tradable/home bias in the consumer's preferences.

Each consumer is endowed with one unit of time that they supply inelastically to the labor market, and the consumer's problem is to maximize C (equation 1) subject to the budget

[2]The small open economy framework is a limiting case (when the relative size of the home country goes to zero) of the standard two-country framework. For brevity, I omit presenting the version of the model where the home country has market power. As examples, both Sutherland (2005) and Alvarez and Lucas (2007) discuss taking the SOE limit of the two country model.

constraint

$$PC = P_{NT}C_{NT} + P_{TR}C_{TR} \quad = \quad w \tag{2}$$

where P_{NT} and P_{TR} are the ideal price indices of the non-tradable and tradable composite goods respectively.

From the consumer's optimizing behavior, the optimal choices of C_{NT} and C_{TR} are given by

$$C_{NT} \quad = \quad \omega \left(\frac{P_{NT}}{P} \right)^{-\eta} C \tag{3}$$

$$C_{TR} \quad = \quad (1-\omega) \left(\frac{P_{TR}}{P} \right)^{-\eta} C \tag{4}$$

where P is the ideal price index of the aggregate consumption good, given by

$$P \quad = \quad \left[\omega P_{NT}^{1-\eta} + (1-\omega) P_{TR}^{1-\eta} \right]^{1/(1-\eta)} \tag{5}$$

3.1.1 Non-tradable sector

The composite non-tradable good (C_{NT}) is produced by aggregating a continuum of individual non-tradable goods indexed over the range $s \in [0, 1]$ using a CES, Dixit-Stigliz, aggregation technology

$$C_{NT} \quad = \quad \left[\int_0^1 c_{NT}(s)^{(\sigma-1)/\sigma} ds \right]^{\sigma/(\sigma-1)} \tag{6}$$

where $c_{NT}(s)$ denotes the quantity of non-tradable good s consumed by an individual consumer, and σ is the elasticity of substitution between any two individual non-tradable goods.

Each good s is the home country's non-tradable sector is produced by perfectly competitive firms that combine labor with a good s specific level of productivity ($z(s) \in \mathbb{R}_+$) using the linear production function

$$y_{NT}(s) \quad = \quad z(s) l_{NT}(s)$$

where $y_{NT}(s)$ is the quantity of output of good s, and $l_{NT}(s)$ is the amount of labor used in the production process.

Because each good $c_{NT}(s)$ enters the composite non-tradable good (C_{NT}) symmetrically, and each good is produced in a perfectly competitive environment, it is possible to group goods by the level of productivity employed in the production process. Doing so allows the expression for the non-tradable composite good to be rewritten as

$$C_{NT} = \left[\int_{\mathbb{R}_+} c_{NT}(z)^{(\sigma-1)/\sigma} f_{NT}(z,t)dz\right]^{\sigma/(\sigma-1)}$$

where $c_{NT}(z)$ is the consumption of all goods $s \in [0,1]$ that are produced using productivity level z, and $f_{NT}(z,t)$ is the PDF of the productivity distribution that exists in the non-tradable sector at time t. Below it will be more convenient to refer to all goods produced using productivity level z as a single good, "good z".

The optimal choice of $c_{NT}(z)$ in the aggregation process is found by minimizing total expenditure on non-tradable goods

$$\int p_{NT}(z)c_{NT}(z)f_{NT}(z,t)dz$$

subject to obtaining a given level of consumption of the composite good (C_{NT}). The optimal choice is thus given by

$$c_{NT}(z) = \left(\frac{p_{NT}(z)}{P_{NT}}\right)^{-\sigma} C_{NT}$$

where P_{NT} is the ideal price index of the composite non-tradable

$$P_{NT} = \left[\int_{\mathbb{R}_+} p_{NT}(z)^{1-\sigma} f_{NT}(z,t)d(z)\right]^{1/(1-\sigma)}$$

Because non-tradable goods producers are perfectly competitive firms, the price of the non-tradable good z is equal to the marginal cost of production, given by

$$p_{NT}(z) = \frac{w}{z} \quad \forall z \in \mathbb{R}_+$$

Substituting this expression into the ideal price index, we can express the ideal price index of the composite non-tradable good (C_{NT}) as a function of the domestic wage rate and the distribution of productivity in the non-tradable sector of the home country.

$$P_{NT} = w\left[\int_{\mathbb{R}_+} z^{\sigma-1} f_{NT}(z,t)d(z)\right]^{1/(1-\sigma)} \tag{7}$$

3.1.2 Tradable sector

Similar to the composite non-tradable good, the composite tradable good (C_{TR}) is a Dixit-Stigliz aggregate of a continuum of individual tradable goods that are produced by perfectly competitive firms. As before, I shall refer to goods as being index by the level of productivity used in their production. Individual tradable goods ($c_{TR}(z)$) can be produced both in the home country and the foreign country. But, by the assumption that the home country is a small open economy, the share of domestic firms in the home country tradable market is insignificant, and the bundle of goods in the composite tradable good is dominated by overseas produced goods (imports).[3] Therefore, the expression for the composite tradable good in the home country is given by

$$C_{TR} = \left[\int_{\mathbb{R}_+} c_{TR}(z)^{(\nu-1)/\nu} f_F(z,t)dz\right]^{\nu/(\nu-1)}$$

where $f_F(z,t)$ is the PDF of the productivity distribution that exists in the foreign country at time t, and ν is the elasticity of substitution between the different individual tradable goods.

The optimal consumption choice of an individual tradable good ($c_{TR}(z)$) by a home country household is found by minimizing total expenditure on tradable goods subject to obtaining a given level of consumption of the composite tradable good, and is given by

$$c_{TR}(z) = \left(\frac{p_{TR}(z)}{P_{TR}}\right)^{-\nu} C_{TR}$$

where $p_{TR}(z)$ is the price in the home country of the tradable good z, and P_{TR} is the ideal

[3] More formally, as the size of the home country shrinks, the distribution of productivity of home country tradable goods producers shrinks proportionally to the size of the country. This ensure that if one were to take many of the smaller countries and aggregate them back up to the original size, the original country world be recovered. In the limit, the measure of goods for which the home country is the least cost producer of tends to zero given the shrinking of the productivity distribution. Therefore, their market share tends to zero in the limit.

price index such that

$$P_{TR} = \left[\int_{\mathbb{R}_+} p_{TR}(z)^{1-\nu} f_F(z,t)d(z)\right]^{1/(1-\nu)}$$

As discussed above, in the case of a SOE, the bundle of individual goods that goes into the composite tradable good is dominated by foreign country producers. Therefore, the price of an individual tradable good ($p_{TR}(z)$) is determined by the marginal cost of producing the good in the foreign country, and the cost involved in shipping it to the home country for consumption. The price of the tradable good z ($p_{TR}(z)$) is given by

$$p_{TR}(z) = \frac{w_F}{\kappa_{H,F} z}$$

where w_F is the wage rate in the foreign country , and $0 < \kappa_{H,F} \leq 1$ is the iceberg trade cost of shipping to the home country from the foreign country. Iceberg trade costs are expressed in a form such that for each unit of good shipped from the foreign country to the home country, $\kappa_{H,F}$ units arrive.

Using this expression for the price of an individual tradable good, the ideal price index of the composite tradable good in the home country can be expressed as

$$P_{TR} = \frac{w_F}{\kappa_{H,F}} \left[\int_{\mathbb{R}_+} z^{\nu-1} f_F(z,t)dz\right]^{1/(1-\nu)} \tag{8}$$

And the price of the composite tradable good is pinned down externally by the foreign wage, trade costs, and the distribution of foreign productivity.

3.1.3 Market clearing

The market for non-tradable goods clears if for each good z in the non-tradable market, the output is equal to demand

$$y_{NT}(z) = c_{NT}(z) \quad \forall z \in \mathbb{R}_+$$

The market for tradable goods clears if for each good z in the tradable market, the output that arrives in the home country is equal to demand

$$\kappa_{H,F} y_{TR}(z) \;=\; c_{TR}(z) \quad \forall z \in \mathbb{R}_+$$

The home country's labor market clears when excess labor demand ($E_H(w, w_F)$) is equal to zero. The expression for the excess labor demand is derived in appendix A using the assumption of balanced trade and is given by

$$E_H(w, w_F)/\tilde{L} \;=\; w_F(1-\omega)\left(\frac{P_{TR}^F}{P^F}\right)^{1-\eta} \hat{D}_{F,H,t} + w\omega\left(\frac{P_{NT}}{P}\right)^{1-\eta} - w \tag{9}$$

where $\tilde{L}$ is a constant, P_{TR}^F is the price index of tradable goods in the foreign country, P^F is the price index of aggregate consumption in the foreign country, and $\hat{D}_{F,H,t}$ is a function of the home country exporter's market share in foreign markets at time t. The first component on the RHS of equation 9 relates to the labor demand to satisfy the production of export goods sold in the foreign country. The second component of the RHS relates to the labor demand required to satisfy the production of non-tradable goods. And the final component of the RHS relates to the supply of labor.

$\hat{D}_{F,H,t}$ is a function of relative wage costs, productivity, and country size between the home and foreign countries, and can be expressed as

$$\hat{D}_{F,H,t} \;=\; \frac{L_F}{\tilde{L}} \lim_{n\to\infty} \int_0^\infty \frac{w}{\kappa_{F,H} p^2} f_{TR}\left(\frac{w}{\kappa_{F,H} p}, t\right) F_{TR}\left(\frac{w}{\kappa_{F,H} p}, t\right)^{-1} F_F\left(\frac{w_F}{p}, t\right) dp$$

where $f_{TR}(z,t)$ and $F_{TR}(z,t)$ are the PDF and CDF of the distributions of productivity in the home country's tradable sector, $\kappa_{F,H}$ is the iceberg trade cost of shipping from the home country to the foreign country, and $L_F/\tilde{L}$ is a constant related to the size of labor forces.

Definition 1: Give a foreign wage rate w_F, A *static trade equilibrium for the home country* is defined as a wage rate $w \in \mathbb{R}_+^n$ such that $E_H(w, w_F)/\tilde{L} = 0$

3.2 Foreign Country

The foreign country interacts with the home country via three channels. First, it produces all of the tradable goods that the home country imports. Second, it provides an export market for the home country firms to sell to so that the home country and earn enough revenue to pay for the imports each period. And third, it provides an external source of productivity ideas for the home country to learn from. The first two of these channels relate to the static trade model, and the third channel related to the productivity diffusion channel discussed in the next section.

For simplicity I model the foreign country using the exact same two-sector structure as the home country. For brevity, I relegate the foreign country static trade model equations to appendix B. The only differences between the home and foreign countries is that the foreign country is assumed to be sufficiently large relative to the home country that it can be treated as a closed economy. Thus both the tradable and non-tradable goods in the foreign country are supplied by the foreign firms.[4]

I also assume that the the foreign country is on the BGP at all points int time, and thus the two sectors of the foreign country share the same distribution of productivity, $F_F(z, t)$. This productivity represents the productivity frontier that exists at time t. This is discussed more below in section 4.

Because the two countries share the same structure, the foreign country represents the BGP limit of the home country (assuming symmetrical iceberg trade costs). Thus, if the home country was able to obtain the level of productivity of the foreign country, the level of output per capita in the home country would converge/catch-up to the level in the foreign country.

4 Productivity Diffusion in the Model

Having established the relationship between the two countries in the static trade model in section 3, I next introduce the dynamic or inter-temporal element into the model. Within

[4] The home country does export to the foreign country as shown in the equation for the excess labor demand in the home country. But because the home country is insignificantly small relative to the foreign country, the market share of home country produces is insignificant from the perspective of the foreign country and thus can be ignored when solving for the equilibrium in the foreign country.

the model, the growth of both countries is driven by the evolution of the productivity distributions for the non-tradable sector ($F_{NT}(z,t)$), tradable sector ($F_{TR}(z,t)$), and the foreign country ($F_F(z,t)$). Following the approaches in Alvarez et al. (2008), and Alvarez et al. (2014), I model the evolution of these productivity distributions as learning for a flow of new ideas.

Think of each firm in the economy as being run by a manager. The manager of each firm is assumed to engage in a search process whereby they meet other managers and exchange production-related ideas. This process is assumed to be a costless process for the managers, and all firms that operate in each market take part in the process.[5] Managers meet a fixed number of other manager each period. If the manager meets another manager from a firm that has a higher level of productivity than theirs, the first manager is able to adopt the productivity practices of the second manager and there by increase the productivity of their firm. If a manager meets another manager whose firm is less productive, the first manager keeps his current production process in place, and productivity is unchanged for his firm. Through this process, higher levels of productivity are able to diffuse among all firms over time.

This diffusion process does not clearly fall into the distinction between imitation and innovation that is often used to describe learning based improvements of productivity (see (König et al., 2012) as an example). On one hand the diffusion process could be considered imitation as firms are merely copying the practices of existing firms and not generating their own new ideas. But on the other hand, because there is a continuum of goods, the diffusion process implies productivity ideas learned from a firm producing a certain good are applied to the production process for other goods. This can be considered innovating in the production process of the other goods. Either interpretation is consistent with the diffusion process in this model and only affects how one describes the diffusion story.

As a result of the diffusion of productivity ideas between firms, the distributions of productivity in the country improves over time, leading to economic growth. Below, I formalize this process as differential equations starting with foreign productivity as this process is independent of the productivity levels in the home country, by the SOE structure of the model.

[5] Therefore managers do not enter the utility function of each country, and no output of firms must be given up to search for new productive ideas.

4.1 Foreign productivity

Let $F_F(z,t)$ denote the CDF of productivity in the foreign country at time t. To describe the law of motion of productivity in the foreign country I begin with the discrete change between period t and period $t+h$, and then derive the continuous-time law of motion as the limiting case.

As a by-product of operating in the foreign market, managers of each foreign firm meet α number of other foreign managers each period.[6] These meetings occur randomly, and independently. The CDF of productivity in the foreign country at time $t+h$ can be expressed as

$$\begin{aligned} F_F(z,t+h) &= Pr\left\{\tilde{z} < z \text{ at time } t+h\right\} \\ &= Pr\left\{\tilde{z} < z \text{ at time } t\right\} \times Pr\left\{\text{all } \alpha h \text{ meetings} \leq z\right\} \\ &= F_F(z,t) \times F_F(z,t)^{\alpha h} \end{aligned}$$

So the fraction of firms with productivity less than z at time $t+h$ depends upon the fraction of firms with productivity less than z at time t, $F_F(z,t)$ (since firms with higher productivity never lose it, the fraction cannot increase over time) and also the probability that all αh meetings do not yield a meeting with another manager whose firm has a productivity greater than z, $F_F(z,t)^{\alpha h}$, from which the firm could learn from and thereby increase their own productivity to a level greater than z.

It then follows that

$$\frac{F_F(z,t+h) - F_F(z,t)}{F_F(z,t)h} = \frac{F_F(z,t)^{\alpha h} - 1}{h}$$

Taking the limit of this expression as $h \to 0$ we have that

$$\frac{\partial \log[F_F(z,t)]}{\partial t} = \alpha \log[F_F(z,t)] \tag{10}$$

which describes the law of motion for the continuous time evolution of the productivity distribution in the foreign country.

[6] Because the distribution of productivity in both the tradable and non-tradable sectors of the foreign country are identical, it does not matter which sector the manager one meets are from.

Given an initial distribution of productivity $F_F(z,0)$, equation 10 implies that the path of $\log\left[F_F(z,t)\right]$ evolves according to

$$\log[F_F(z,t)] \quad = \quad \log[F_F(z,0)]e^{\alpha t} \tag{11}$$

Because the foreign country represents the productivity frontier and is unaffected by developments in the home country, it is convenient to think about the foreign country as being on a sustained balanced growth path (BGP). I next turn to characterizing the balance growth path of the foreign country and the initial distributions that will lead to the distribution of productivity converging asymptotically to the balanced growth path over time.

Let the balanced growth path of the foreign country be defined as a CDF of relative productivity $\Psi_F(z)$ and a growth rate $g > 0$ such that

$$F_F(e^{gt}z,t) \quad = \quad \Psi_F(z) \quad \forall t$$

Proposition 1: The CDF and growth rate pair $\{\Psi_F(z), g\}$ is a balanced growth path of equation 10 if and only if $\Psi_F(z)$ is a Fréchet distribution with parameters $\lambda_F > 0$ and $\theta_F = g/\alpha > 0$ defined by

$$\Psi_F(z) \quad = \quad \exp\left(-\lambda_F z^{-1/\theta_F}\right)$$

Proof: If $F_F(z,0)$ is a Fréchet distribution with parameters (λ_F, θ_F), equation 11 implies that $F_F(z,t)$ is distributed Fréchet with parameters $(\lambda_F e^{\alpha t}, \theta_F)$. And $\Psi_F(z) = F_F(e^{gt}z,t)$ implies that if $F_F(z,t)$ is distributed Fréchet, so is $\Psi_F(z)$.

Conversely, suppose (Ψ_F, g) is a balance growth path such that $F_F(z,t) = \Psi_F(e^{-gt}z)$ solves equation 11. Then

$$\log[\Psi_F(e^{-gt}z)] \quad = \quad \log[\Psi_F(z)]e^{\alpha t}$$

Differentiating this expression with respect to t and evaluating at $t = 0$ yields the following

Riccati equation

$$-\frac{\Psi_F'(z)}{\Psi_F(z)}\theta z = \log[\Psi_F(z)]$$

which has the general solution

$$\Psi_F(z) = \exp\left[kz^{-1/\theta_F}\right]$$

where k is a constant. A change of variable ($k = -\lambda_F$) completes the proof.

From proposition 1, we learn that along the BGP the distribution of relative productivity is Fréchet and growing at a constant rate g. The Fréchet distribution is governed by two parameters. λ_F is often referred to as the scale parameter. A larger value of λ_F increases (shifts right) the entire distribution of productivity. So a country who has a productivity distribution with a larger scale parameter, all else equal, will have a absolute advantage relative to a country whose productivity distribution has a lower scale parameter. The parameter θ is often referred to the shape parameter. A larger value of θ implies a more disperse distribution of productivity (a distribution with a fatter tail). In the model, θ_F relates to the degree of comparative advantage within a sector.

Next I turn to examining what types of initial distributions imply asymptotic convergence of the productivity distribution to the balanced growth path. More formally, we seek conditions on $F_F(z, 0)$ to ensure that

$$\lim_{t\to\infty} \log[F_F(e^{gt}z, t)] = -\lambda_F z^{-1/\theta_F} \tag{12}$$

Proposition 2: The solution in equation 11 to equation 10 satisfies the above stability condition (equation 12) for some λ_F and θ if and only if the initial distribution $F_F(z, 0)$ satisfies the following condition

$$\lim_{z\to\infty} \frac{1 - F_F(z, 0)}{z^{-1/\theta_F}} = \lim_{z\to\infty} \theta_F z^{1/\theta_F+1} f_F(z, 0) = \lambda_F$$

This condition implies that the initial distribution has a Pareto tail.

Proof : Combine equation 11 with equation 12 yields

$$\lim_{t\to\infty} \frac{\log[F_F(e^{gt}z,0)]e^{\alpha t}}{z^{-1/\theta_F}} \;=\; -\lambda_F$$

Using change of variable, $x \equiv e^{g_F t}z$, this is equivalent to

$$\lim_{x\to\infty} \frac{\log F_F(x,0)}{x^{-1/\theta_F}} \;=\; -\lambda_F$$

And using the fact that $\lim_{x\to\infty}[-\log F_F(x,0)] = \lim_{x\to\infty}[1-F(x,0)]$ we have that

$$\lim_{x\to\infty} \frac{1-F_F(x,0)}{x^{-1/\theta_F}} \;=\; \lambda_F$$

which is equivalent to the required condition on the initial distribution.

The converse also holds. This can be shown by applying the above steps in reverse order.

From propositions 1 and 2, if the initial distribution distribution has a Pareto tail, it will asymptotically converge to the BGP that is Fréchet in distribution. And since the Fréchet distribution has a Pareto tail, if the initial distribution is Fréchet, the distribution of productivity in the foreign country will begin on the BGP, and stay on the BGP, maintaining its Fréchet shape. In the remainder of this paper, I will limit the analysis to this case as my focus is not on the dynamics of the foreign economy off the BGP.

4.2 Non-tradable productivity

Managers of non-tradable firms in the home country are assumed to meet other non-tradable managers at rate ρ per period, and also meet managers of firms that operate in the home country's tradable sector at rate β. As outlined in the static trade model section above, home country tradable firms hold an insignificant share of the home country's tradable market. And because meetings with firms within a sector are based upon their market share (the number of goods they produce), the probability of a non-tradable firm meeting a home country tradable firm is zero. Thus, the CDF of productivity in the non-tradable sector of the home country at time $t+h$ can be expressed as

$$\begin{aligned} F_{NT}(z,t+h) &= Pr\{\tilde{z} < z \text{ at time } t+h\} \\ &= Pr\{\tilde{z} < z \text{ at time } t\} \times Pr\{\text{all } \rho h + \beta h \text{ meetings} \leq z\} \\ &= F_{NT}(z,t) \times \left[F_{NT}(z,t)^{\rho h} F_F(z,t)^{\beta h}\right] \end{aligned}$$

So the fraction of firms in the non-tradable sector with productivity less than z at time $t+h$ depends upon the initial fraction of firms with productivity less than z at time t, $F_{NT}(z,t)$, and also the probability that all the independent ρh meetings with other non-tradable managers and βh meetings with foreign/tradable managers do not yield another manager with a productivity greater than z from which the firm could learn from, $F_{NT}(z,t)^{\rho h} F_F(z,t)^{\beta h}$.

It then follows that in the continuous time limit of this expression ($h \to 0$) we have that

$$\frac{\partial \log[F_{NT}(z,t)]}{\partial t} = \rho \log[F_{NT}(z,t)] + \beta \log[F_F(z,t)] \tag{13}$$

which describes the law of motion for the continuous time evolution of the productivity distribution in the non-tradable sector of the home country.

Given an initial distribution of productivity in the non-tradable sector $F_{NT}(z,0)$, and tradable sector $F_F(z,0)$, equation 13 implies that the path of $\log[F_{NT}(z,t)]$ evolves according to

$$\log[F_{NT}(z,t)] = \frac{\beta}{\alpha-\rho} \log[F_F(z,0)] \left[e^{\alpha t} - e^{\rho t}\right] + e^{\rho t} \log[F_{NT}(z,0)] \tag{14}$$

where I have used the fact that $F_F(z,t)$ evolves independently over time according to the process outlined in equation 11.

Because this paper is focused on deriving a model of income convergence (catchup growth) our attention must be restricted to the case where the BGP of the non-tradable sector is the same as the foreign country. Therefore, I redefine $\Psi \equiv \Psi_F$ as the CDF in the BGP $\{\Psi, g\}$ common to both countries. Next I examine what conditions on $F_{NT}(z,0)$ and $F_F(z,0)$ implies convergence of the home country's non-tradable productivity distribution to the BGP. Convergence to the BGP can be expressed as the following stability condition for the non-tradable sector

$$\lim_{t\to\infty} \log\left[F_{NT}\left(e^{gt}z,t\right)\right] \;=\; -\lambda_F z^{-1/\theta_F} \equiv \log\left[\Psi(z)\right] \tag{15}$$

Proposition 3: The solution in equation 14 to equation 13 satisfies the above stability condition (equation 15) if the following conditions are meet

$$\begin{aligned}
\lim_{z\to\infty} \frac{1-F_{NT}(z,0)}{z^{-1/\theta_{NT}}} &= \lambda_{NT} < \infty \\
\lim_{z\to\infty} \frac{1-F_F(z,0)}{z^{-1/\theta_F}} &= \lambda_F < \infty \\
\alpha &= \rho + \beta \\
\theta_F &\geq \theta_{NT}
\end{aligned}$$

The first two conditions state that the initial distributions of productivity in the non-tradable sector and foreign country have Pareto tails. The third condition states that the aggregate rate at which agents meet other agents, of any type, is the same in both countries. The final condition states that the distribution of productivity in the foreign country has a weakly thicker tail (is more right skewed) than the distribution of non-tradable productivity.

Proof: Using equation 14, the non-tradable stability condition (equation 15) can be re-expressed as

$$\lim_{t\to\infty} \left\{ \frac{\beta}{\rho-\alpha} \log[F_F(e^{gt}z,0)] \left[e^{\rho t} - e^{\alpha t}\right] + e^{\rho t} \log[F_{NT}(e^{gt}z,0)] \right\} \;=\; -\lambda_F z^{-1/\theta_F}$$

Rearranging this equation and preforming a change of variables ($x = e^{g_{NT}t}z$), yields the condition

$$\begin{aligned}
&\frac{\beta}{\rho-\alpha} \lim_{x\to\infty} \left\{ \frac{1-F_F(x,0)}{x^{-1/\theta_F}} \right\} \left[1 - \lim_{t\to\infty}\left\{e^{(\rho-\alpha)t}\right\}\right] \ldots \\
&- \lim_{t\to\infty}\left\{e^{(\rho-\alpha)t}\right\} \lim_{x\to\infty} \left\{ \frac{1-F_{NT}(x,0)}{x^{-1/\theta_{NT}}} \right\} \lim_{x\to\infty} \left\{ \frac{x^{-1/\theta_{NT}}}{x^{-1/\theta_F}} \right\} \;=\; -\lambda_F
\end{aligned} \tag{16}$$

Using the conditions that the initial distributions $F_F(x,0)$ and $F_{NT}(x,0)$ have Pareto tails, equation 16 becomes

$$\frac{\beta}{\rho-\alpha}\lambda_F\left[1-\lim_{t\to\infty}\left\{e^{(\rho-\alpha)t}\right\}\right]-\lambda_{NT}\lim_{t\to\infty}\left\{e^{(\rho-\alpha)t}\right\}\lim_{x\to\infty}\left\{\frac{x^{-1/\theta_{NT}}}{x^{-1/\theta_F}}\right\} \;=\; -\lambda_F$$

Given that $\theta_F \geq \theta_{NT}$

$$\lim_{x\to\infty}\left\{\frac{x^{-1/\theta_{NT}}}{x^{-1/\theta_F}}\right\} \;=\; \begin{cases} 0 & \text{if } \theta_F > \theta_{NT} \\ 1 & \text{if } \theta_F = \theta_{NT} \end{cases}$$

$\alpha > \rho$ implies

$$\lim_{t\to\infty}\left\{e^{(\rho-\alpha)t}\right\} \;=\; 0$$

Using these facts, equation 16 simplifies down to

$$\frac{\beta}{\rho-\alpha}\lambda_F \;=\; -\lambda_F$$

which hold given that $\alpha = \rho + \beta$ by the third condition. For the remainder of this paper, I shall assume that the conditions in proposition 3 hold, and the home country's non-tradable productivity distribution converges to the same BGP as the foreign country.

The small open economy assumption of the two-country trade component of this model ensures that the tradable market in both countries is dominated by the distribution of foreign producer firms who sell to the home country. Thus, the evolution of productivity in the model is independent of all other trade related elements (e.g. trade costs, wages, prices, etc) in the model, and depends only upon the distributions of other productivity in the model, and meeting rates. From equation 14, we can study how the Pareto tail of the distribution of non-tradable productivity in the home country ($\lambda_{NT}(t)$) converges to the tail parameter of the foreign country distribution (λ_F) in the limit. The difference in tails between the two distributions is given by

$$\lambda_{NT}(t) - \lambda_F \;=\; -\lambda_F e^{-\beta t}$$

for all $t > 0$. This expression shows that the Pareto tail of the non-tradable productivity in the home country converges exponentially to that of the foreign productivity distribution. And as such, the half life of the tail is given by $\ln(2)/\beta$.

4.3 Tradable productivity

Managers of tradable firms in the home country are assumed to meet managers of tradable firms that operate in the home country (i.e. foreign firms) at rate γ, and meet non-tradable managers at rate δ per period. Thus, the CDF of productivity in the tradable sector of the home country at time $t+h$ can be expressed as

$$\begin{aligned} F_{TR}(z, t+h) &= Pr\left\{\tilde{z} < z \text{ at time } t+h\right\} \\ &= Pr\left\{\tilde{z} < z \text{ at time } t\right\} \times Pr\left\{\text{all } \gamma h + \delta h \text{ meetings} \leq z\right\} \\ &= F_{TR}(z,t) \times \left[F_F(z,t)^{\gamma h} F_{NT}(z,t)^{\delta h}\right] \end{aligned}$$

It then follows that the limit of this expression as $h \to 0$ is given by

$$\frac{\partial \log[F_{TR}(z,t)]}{\partial t} = \gamma \log[F_F(z,t)] + \delta \log[F_{NT}(z,t)] \tag{17}$$

which describes the law of motion for the continuous time evolution of the productivity distribution of home country firms in the tradable sector of the home country.

Given an initial distribution of productivity of the home tradable firms $F_{TR}(z,0)$, non-tradable sector $F_{NT}(z,0)$, and foreign sector $F_F(z,0)$, equation 17 implies that the path of $\log\left[F_{TR}(z,t)\right]$ evolves according to

$$\begin{aligned} \log[F_{TR}(z,t)] &= \log[F_{TR}(z,0)] + \log[F_F(z,0)]\left[\frac{\beta\delta + (\alpha-\rho)\gamma}{\alpha(\alpha-\rho)}\left[e^{\alpha t}-1\right] - \frac{\beta\delta}{\rho(\alpha-\rho)}\left(e^{\rho t}-1\right)\right] \\ &\quad + \frac{\delta}{\rho}\log[F_{NT}(z,0)]\left[e^{\rho t}-1\right] \end{aligned} \tag{18}$$

where I have used the expressions for the evolution of $F_{NT}(z,t)$ and $F_F(z,t)$ found in the two previous subsections.

The productivity of tradable firms converges to the common BGP if the following tradable sector stability condition is satisfied

$$\lim_{t\to\infty} \log\left[F_{TR}\left(e^{gt}z,t\right)\right] \quad = \quad -\lambda_F z^{-1/\theta_F} \equiv \log\left[\Psi(z)\right] \tag{19}$$

Proposition 4: The tradable sector stability condition (equation 19) is satisfied if

$$\begin{aligned} \alpha &= \gamma+\delta \\ \lim_{z\to\infty}\frac{1-F_{TR}(z,0)}{z^{-1/\theta_{TR}}} &= \lambda_{TR}<\infty \\ \theta_F &\geq \theta_{TR} \end{aligned}$$

and proposition 3 holds.

Proof Combining equation 18 with the tradable stability condition yields:

$$\begin{aligned} \log[F_{TR}(e^{gt}z,t)] \quad = \quad & \log[F_{TR}(e^{gt}z,0)]+\log[F_F(e^{gt}z,0)]\left[\frac{\beta\delta+(\alpha-\rho)\gamma}{\alpha(\alpha-\rho)}\left[e^{\alpha t}-1\right]-\frac{\beta\delta}{\rho(\alpha-\rho)}\left(e^{\rho t}-1\right)\right] \\ & +\frac{\delta}{\rho}\log[F_{NT}(e^{gt}z,0)]\left[e^{\rho t}-1\right] \end{aligned}$$

Therefore, the stability condition is satisfied if

$$\lim_{t\to\infty}\left\{\begin{array}{l} \log[F_{TR}(e^{gt}z,0)]+\log[F_F(e^{gt}z,0)]\left[\frac{\beta\delta+(\alpha-\rho)\gamma}{\alpha(\alpha-\rho)}\left[e^{\alpha t}-1\right]-\frac{\beta\delta}{\rho(\alpha-\rho)}\left(e^{\rho t}-1\right)\right] \\ +\frac{\delta}{\rho}\log[F_{NT}(e^{gt}z,0)]\left[e^{\rho t}-1\right] \end{array}\right\} \quad = \quad -\lambda_F z^{-1/\theta_F}$$

Using a change of variable ($x=e^{gt}z$) and the fact $\lim_{x\to\infty}\log F(x,0)=-\lim_{x\to\infty}[1-F(x,0)]$

$$\begin{aligned} & \lim_{x\to\infty}\left\{\frac{1-F_{TR}(x,0)}{x^{-1/\theta_{TR}}}\right\}\lim_{x\to\infty}\left\{\frac{x^{-1/\theta_{TR}}}{x^{-1/\theta_F}}\right\}\lim_{t\to\infty}e^{-\alpha t} \\ + \lim_{x\to\infty}\left\{\frac{1-F_F(x,0)}{x^{-1/\theta_F}}\right\}\lim_{t\to\infty}e^{-\alpha t} & \left[\frac{\beta\delta+(\alpha-\rho)\gamma}{\alpha(\alpha-\rho)}\left[e^{\alpha t}-1\right]-\frac{\beta\delta}{\rho(\alpha-\rho)}\left(e^{\rho t}-1\right)\right] \quad = \quad \lambda_F \\ & +\frac{\delta}{\rho}\lim_{x\to\infty}\left\{\frac{1-F_{NT}(x,0)}{x^{-1/\theta_F}}\right\}\lim_{t\to\infty}\left[e^{(\rho-\alpha)t}-e^{-\alpha t}\right] \end{aligned}$$

Applying proposition 3, we have

$$\left[\frac{\delta+\gamma}{\alpha}\left[e^{\alpha t}-1\right]-\frac{\delta}{\rho}\left(e^{\rho t}-1\right)\right]$$

$$\lim_{x\to\infty}\left\{\frac{1-F_{TR}(x,0)}{x^{-1/\theta_{TR}}}\right\}\lim_{x\to\infty}\left\{\frac{x^{-1/\theta_{TR}}}{x^{-1/\theta_{F}}}\right\}\lim_{t\to\infty}e^{-\alpha t}+\lambda_F\left[\frac{\delta+\gamma}{\alpha}\left[1-e^{-\alpha t}\right]-\frac{\delta}{\rho}\left(e^{(\rho-\alpha)t}-e^{-\alpha t}\right)\right] = \lambda_F$$

And applying the conditions in proposition 4

$$\lambda_F\left[\frac{\delta+\gamma}{\alpha}\right] = \lambda_F$$

Given the assumption that $\alpha=\gamma+\delta$, the convergence condition is satisfied.

The gap between the Pareto tail of the home country's tradable productivity distribution and the Pareto tail of the foreign country is given by

$$\lambda_{TR}(t)-\lambda_F = -\lambda_F e^{-\alpha t}-\frac{\delta}{\rho}\lambda_F\left(e^{-\beta t}-e^{-\alpha t}\right)$$

Note that the above equation is a weighted sum of two exponential processes, and hence itself does not an exponential process. Thus, the half life of the process is not independent of time. However, in the limiting case where the home country tradable sector does not learn from the non-tradable sector, $\delta=0$, and $\gamma=\alpha$ and we have

$$\lambda_{TR}(t)-\lambda_F = -\lambda_F e^{-\alpha t}$$

which has a half-life of $\ln(2)/\alpha$.

Combining the laws of motion that describe the productivity diffusion of the foreign country, and both the non-tradable and tradable sectors of the home country with the static trade model completes the entire model. Given the initial distributions of productivity, the laws of motion provide a deterministic path for the future productivity distribution that converges to the same BGP in the limit, implying catch up growth for the home country. And at each point in time, the static trade model equations convert the distribution of productivity into measures of income for the home country. In the remainder of this paper I calibrate the model and examine the predict paths of output growth from the model.

5 Calibration

In this section I discuss the calibration of the model that will be used to simulate the results found in section 6. Table 1 summarizes the parameter values calibrated to match typical values found in the literature. And table 2 details the parameters of the model calibrated to match particular moments in the data.

The panel of countries featured in figure 1 is a very diverse set with regards to the composition of each country's industries, trade policies, endowments, etc. The model that I use in this paper is not designed with enough richness in structure to accurately capture the idiosyncratic nuances of each country. Rather than complicate the model in an attempt to fit the data from all countries, I instead aim to present a calibration that captures the common features of developing open countries where possible, without matching any specific country. Therefore, my focus is squarely on how far the productivity diffusion story get us towards matching the data's broadly patterns.

5.1 Calibrated parameters

According to Alvarez et al. (2014), the range of values in the literature for the elasticity of substitution between individual goods is $\eta, \sigma \in [3, 10]$. De Paoli (2009) uses a value of 10 in her model, and Gali and Monacelli (2005) uses a value of 6. Within the structure of my model, the range of possible values for the elasticity of substitution within a sector is restricted by the productivity distribution shape parameter θ. If θ is large, the distribution of productivity in the sector has a 'fat tail' with a large measure of firms that are extremely productive. And if goods within the sector are "too substitutable", that is η or σ is large, Then labor will only be allocated to the most productive firms and the integral in the CES aggregation functions for C_{TR} and C_{NT} is undefined (infinite). Thus, for the model to be well defined for a plausible values of θ (discussed below), I set $\eta = \sigma = 5$, which is still within the range of values used in the literature.

The calibration of the elasticity of substitution between non-tradable (home) goods and tradable (foreign) goods used in the literature is generally lower than the elasticity of substi-

Table 1: Calibrated parameter values

Trade Related Parameters

Parameter	Description	Value
σ	Elasticity of sub. b/w different NT goods	5
ν	Elasticity of sub. b/w different tradable (foreign) goods	5
η	Elasticity of sub. between tradable and non-tradable	3
$\kappa_{F,H}$	Iceberg trade cost (from home to foreign country)	0.75
$\kappa_{H,F}$	Iceberg trade cost (from foreign to home country)	0.75

Growth Related Parameters

Parameter	Description	Value
θ_F	Shape parameter foreign productivity	0.2
θ_{NT}	Shape parameter home country non-tradable productivity	0.2
θ_{TR}	Shape parameter home country tradable productivity	0.2

tution between individual tradable or non-tradable goods. Obstfeld and Rogoff (2000) use a value of $\eta = 6$ as their baseline for the elasticity of substitution between non-tradable goods and tradable goods based on recent trade studies, which generally find values in the range 5 to 6. De Paoli (2009) uses a value of of 3. I choose to use a value of 3 implying that the composite non-tradable and tradable goods are less substitutable in the final consumption good than the individual goods that go into making each composite good.

Following Alvarez and Lucas (2007), I set the both trade cost parameters equal to 0.75 ($\kappa_{H,F} = \kappa_{F,H} = 0.75$). This value reflects a balance between estimates of trade costs via gravity equations, that support values as low as 0.65, and direct estimates of trade costs that support values around 0.9. By assuming the trade costs are symmetrical, the model structure implies that the foreign country can be consisted to be a limiting case of the home country on the balanced growth path.

Lucas (2009a) estimates the shape parameter related to the productivity distribution to be $\theta = 0.5$ using data on earnings from the 1990 US census. Alvarez et al. (2014) discuss various other approaches available to estimate the shape parameter of productivity (θ). The shape parameter estimated by the various approaches lie in the range of $\theta \in [0.1, 0.6]$. They choose to calibrate $\theta = 0.2$, which lies in the range of estimates suggested by the Armington trade elasticity. I choose to calibrate the tail parameter of the foreign country to be $\theta_F = 0.2$,

Table 2: Parameters to match data moments

Parameter	Description	Targeted moment
ω	Home bias in consumption	Import to GDP ratio on the BGP
λ_F	Scale parameter foreign productivity	GDP per capita in foreign country at time $t = 0$
λ_{NT}	Scale parameter non-tradable productivity	Initial import to GDP and C/C_F ratios at time $t = 0$
λ_{TR}	Scale parameter tradable productivity	Initial import to GDP and C/C_F ratios at time $t = 0$
α	Meeting rate between foreign and foreign firms	growth rate of GDP per capita on the BGP
ρ	Meeting rate between non-tradable and non-tradable firms	Convergence rate in non-tradable industries
β	Meeting rate between non-tradable and tradable firms	Convergence rate in non-tradable industries
γ	Meeting rate between tradable and foreign firms	Convergence rate in tradable industries
δ	Meeting rate between tradable and non-tradable firms	Convergence rate in tradable industries

which lies at the upper end of the range of feasible values for the model given that $\eta = 5$.

I choose to set the tail parameter for the home country's non-tradable and tradables firms to be $\theta_{NT} = \theta_{TR} = 0.2$. Setting the tail parameter in the home country to be the same as the foreign country implies that if the home country does not open up to trade and learning from the foreign country, its will converge to a balanced growth path that grows at 2% annually, but at a lower level than the foreign country. The distribution of growth rates for closed countries in figure 1 is very disperse and 2% falls close to the middle of the range.

5.2 Parameters to match data moments

I now turn to the parameter values chosen to match particular moments in the data. The parameters and their intended targets are outline in table 2. I choose the value of ω (the home bias in consumption) to match the long-run average of imports to GDP on the BGP. The targeted value of the import share is discussed below.

The value of λ_F is chosen so that the foreign country has GDP per capita equal to $\$20,000$, in 1995 PPP terms, in the year 1960 (see figure 1).

The parameters λ_{TR} and λ_{NT} are chosen to fit two moments jointly. First the initial level of GDP per capita in the home country in the year 1960. And second, the initial import ratio (C_{TR}/C) in 1960.

The targeted moments of the initial import to GDP ratio for the home country and the long-run BGP import to GDP ratio (for the foreign country) are found using data on the import share of open countries in the Penn World Tables 8.0. The import share of countries varies significantly across countries. Countries such as Spain, Indonesia, and the United states have import shares 15 percent, while countries such as Belgium, Hong Kong, Norway, and Singapore have import shares in excess of 100 percent. However, for the purposes of calibrating this model, I am interested in a "typical" small open economy. To derive the import share of this theoretical "typical" country in the initial period, I regress import share onto GDP per capita (for observations with GDP per capita up to 20,000 – the initial level of the foreign country) and a country specific fixed effect. I then take the median country specific fixed effect value and the coefficient on GDP per capita to construct a fitted value for the import share of the "typical" small open economy.

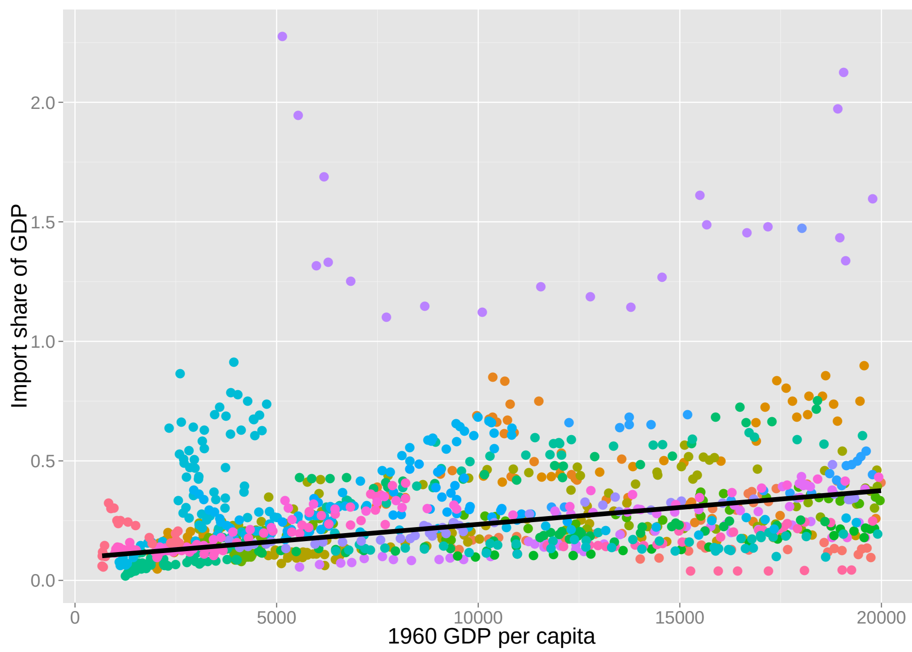
Figure 2 plots the fitted value line against the import share data for the open countries in my panel. The fitted values imply that the foreign country has an import to GDP ratio of 0.38 (close commonly calibrated values in the literature), and a poor country that has an initial GDP per capita level of 2,000 has an import to GDP ratio of around 0.12.

The rate at which foreign firms meet other foreign firms (α) is calibrated so that the foreign country grows at an annual rate of two percent on its BGP, matching the values implied by figure 1.

5.2.1 Home country meeting rates

It is difficult to map the meeting rates between firms to a specific real world concept. In reality, there is a selection effect that biases the set of firms that a particular firm will interact with the most during the course of business. Firms in other parts of the integrated supply chain, and also direct competitors are more likely to be interacted with or monitored more by an individual firm than firms in other sectors. In addition, a more productive idea or process from a foreign firm or a firm in a different industry may be harder to implement

Figure 2: Import share of GDP for open countries



The points in the graph above refer to annual observations for each country in the sample. The points are colored according to their country. The regression line shown above is found by regressing import share onto GDP per capita and a country fixed effect for observations with GDP per capita less than 20,000. The fitted line is constructed using the median country fixed effect and slope parameter from the regression. It shows that richer countries have on average a larger import to GDP ratio.

than a more productive idea from the same industry for a particular firm.

Therefore, I view the meeting rates within the model as reduced form parameters. It is a parameter to represent a complicated function of how firms interact as well as the implementability of ideas across firms from different countries and sectors.

For guidance as to how to calibrate the rate of at which home country firms meet other firms (ρ, β, γ, δ), I use the EU KLEMS Growth and Productivity Accounts database.[7] This database provides information for 32 industries for 30, predominantly European and OECD, countries. From the database, I construct a measure of industry level productivity for each country using data from 1977 to 2007.[8] I define industry level productivity to be the real value added per hour worked in industry-specific PPP terms.

Using the growth and level accounting methods discussed by Inklaar et al. (2008), for each industry-country pair it is possible to construct a measure of industry-level productivity in one country relative to another. I construct a measure of the relative distance between industry level productivity in each country and the level of productivity in the corresponding to the most productive country in each industry, at each point in time, for all industries. To help map the model to the data, I assume that, at each point in time, the most productive country is on the productivity frontier for that industry.

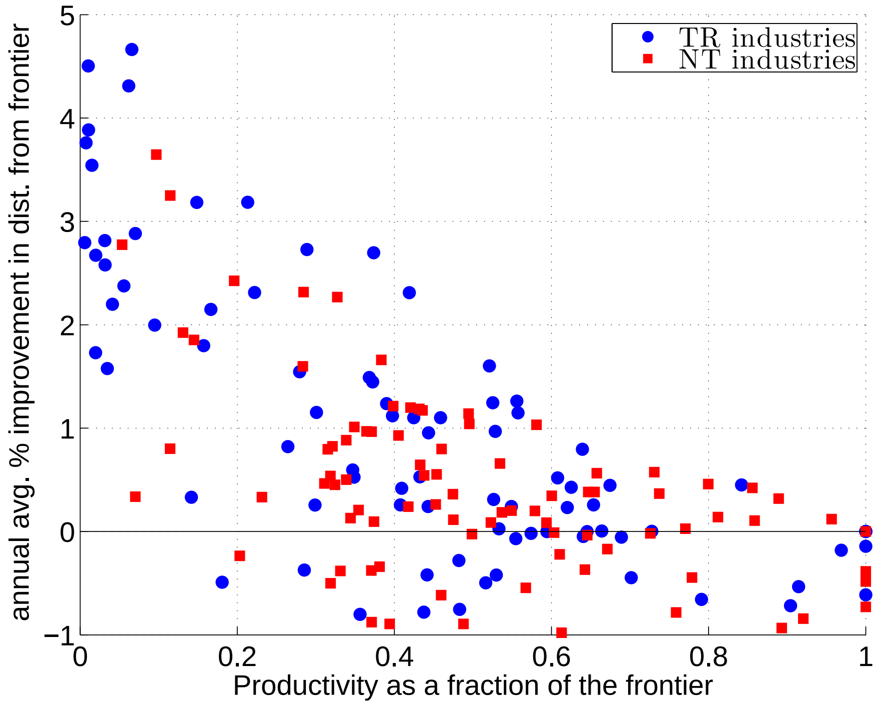
Figure 3 plots the productivity of industry-country pairs for select industries, and also those countries in the EU KLEMS database that Sachs and Warner (1995) defines as open. In addition, I classify each industry as tradable or non-tradable based upon classifications typically employed in the literature.

The scatter plot in figure 3 shows that there is a large degree of hetrogeniety on the productivity growth rates across industry-country pairs. Despite this, there is broad pattern that shows industry-country pairs that started further below the productivity frontier improved more on average than those close to the frontier between 1977 and 2007. Those industry-country pairs that were furthest from the frontier improved their distance from the frontier (get closer to the frontier) at an average rate of around 2 to 5 percent each year. And there does not appear to be any systematic difference between the catch-up rates of productivity improvement between industries that are classified as tradable industries typically in the

[7]Specifically, I use Revision 3 of the database. More details of the database and methodology used in its construction can be found in O'Mahony and Timmer (2009)

[8]The sample length used here is shorter than the previous analysis in this paper to increase the coverage of countries within the subsample I use.

Figure 3: Industry level productivity convergence in open economies (1977-2007)



Industries that are classified as tradable include: Agriculture, hunting, forestry, and fishing; Mining and quarrying; Food, beverages and tobacco; Textiles, leather and footwear; Basic metals and fabricated metal; Machinery NEC; Electrical and optical equipment; Manufacturing NEC. Industries that are classified as non-tradable include: Electricity, gas, and water supply; Construction; Wholesale trade; Retail trade ex. motor vechicles; Hotels and restaurants; Transport and storage; Health and social work. Countries includes in the sample: AUS, AUT, BEL, CYP, DNK, ESP, FIN, FRA, GRC, IRL, ITA, JPN, KOR, NLD, PRT, SWE, and USA.

literature, and non-tradable industries.[9]

Figure 3 also shows that there were a number of industry-country pairs that did not keep up with the productivity frontier in their industry, and decreased in productivity relative to the productivity frontier. These industry-country pairs are shown by the points below the zero line. The model's simple growth process outline in section 4 does not allow for productivity in the home country to decline relative to the foreign country. Therefore, in calibrating the meeting rates, the focus will be on finding values that imply average growth rates in productivity similar to those shown in figure Figure 3.

To assist in interpreting the calibration of the meeting rates, I define a new variable μ_i as the fraction of total meetings in sector $i \in \{TR, NT\}$ that occur with foreign country firms.[10] Therefore, μ_i can be given by

$$
\begin{aligned}
\mu_{NT} &= \frac{\beta}{\rho+\beta} = \frac{\beta}{\alpha} \\
\mu_{TR} &= \frac{\gamma}{\gamma+\delta} = \frac{\gamma}{\alpha}
\end{aligned}
$$

I consider a wide range of values for μ_{NT} and μ_{TR}, from 0.5 percent to 50 percent, I simulate the productivity diffusion process outlined in section 4 for the home country starting at different distances from the productivity frontier (the foreign country's productivity). In this analysis, I focus solely on the non-tradable sector. Like the data plotted in figure 3, the growth rate of non-tradable sector relative to the frontier depends only upon its own productivity distribution, and the distribution at the frontier. Thus the data can be used to calibrate the meeting rates for the non-tradable sector.

Within the model, the growth rate of productivity in the tradable sector depends not only on how far behind the frontier it is, but also upon its relative position to the productivity in the non-tradable sector. Given that the EU KLEMS does not provide information on the exact level of productivity (only relative levels), it is difficult to match the data to both tradable and non-tradable productivity in the home country and foreign productivity.

Given we can estimate the meeting rates for the non-tradable sector, two facts can help

[9] However, there could still be a level difference between the productivity frontiers of each industry not picked up in the figure.

[10] Recall that propositions 3 and 4 require that the meeting rates in each sector of the home country equal the foreign country's meeting rate α. These propositions combined with μ_i is sufficient to identify the individual meeting rates ρ, β, γ, and δ.

us choose appropriate meeting rates for the tradable sector. First, because the tradable and non-tradable firms are learning from the same source distributions, the difference in growth rates between the two sectors is entirely due to the tradable sector starting from a difference base level of productivity. And as equation 18 shows, the effect of the initial level of tradable sector productivity dies out rapidly given the exponential weighting on the non-tradable and foreign productivity distributions. Therefore, if the meeting rates of the tradable and non-tradable sectors are the same, the difference in growth rates between the will be relatively small for long time frames. And second, from figure 3 we have seen that the rate of catch-up growth does not differ significantly between tradable and non-tradable industries. Therefore, similar meeting rates between the two sectors are likely to be suitable for calibrating the model to the data.

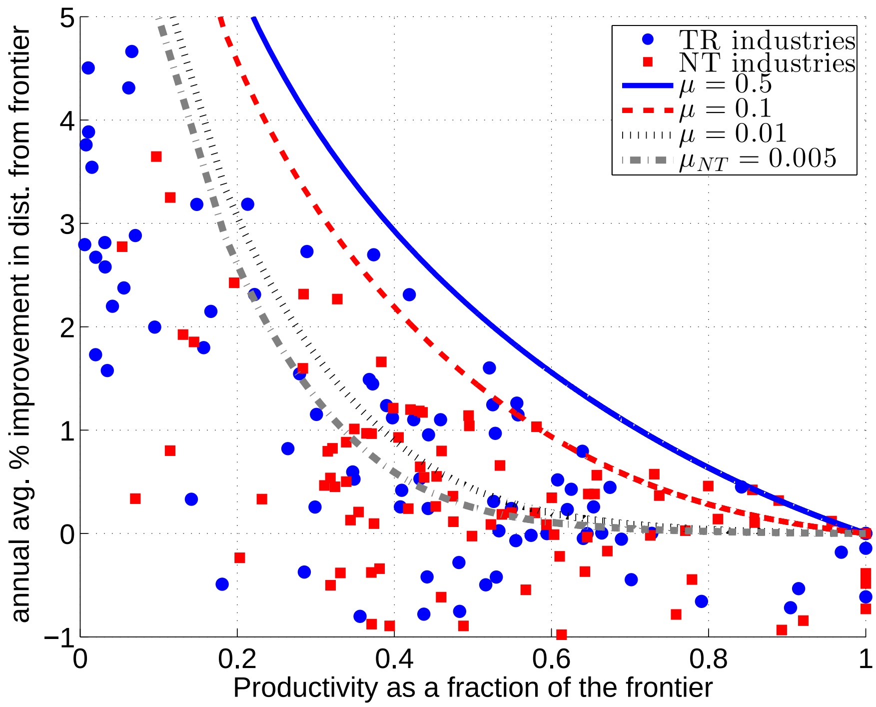
Figure 4 presents the results from simulating the growth in average productivity in the non-tradable sector of the model at various meeting rates and for various distances from the productivity frontier. To broadly match the pattern of catch-up observed in the productivity of industry-country pairs between 1977 and 2007, the model requires a relatively small fraction of the meetings between non-tradable firms and other firms to be with foreign firms. When the fraction of meetings between the home country firms and foreign firms is one percent or below, the growth rates become less sensitive to the share of foreign meetings (μ), and tends to slightly over estimate the growth rate of those countries starting well behind the frontier (initial productivity less than 0.2 of the frontier), and underestimate the growth rate of those firms closer to the frontier (initial productivity greater than 0.5 of the frontier).[11]

For the purposes of calibrating the model, I choose to set $\mu_{NT} = \mu_{TR} = 0.01$. As discussed above, using the same μ parameter for the tradable sector will produce a marginally higher growth rate when the tradable sector started from a lower base (which will be the appropriate case to consider for the results).[12]

[11] In the limit as $\mu \to 0$, the annual average percentage improvement in the distance from the frontier tends to zero in the model. This implies that the home country maintains the sames distance from the frontier for all time periods. This result is predicated on the calibration that $\theta_F = \theta_{NT} = \theta_{TR}$. If the value of θ for the home country was lower than the foreign country, the annual average percentage improvement in the distance from the frontier would tend to a negative value. The thinner tail of the productivity distribution in the home country (lower θ) will hinder the ability of a given firm to meet a more productive firm, and thus the growth rate in productivity ($g_{NT} = \theta_{NT}\rho$ when $\mu_{NT} = 0$) would be lower.

[12] As a test of the sensitivity, I also consider some of the other values for μ_{NT} and μ_{TR} in the results sections as a check on robustness.

Figure 4: Industry level productivity convergence in open economies (1977-2007)



Industries that are classified as tradable (TR) include: Agriculture, hunting, forestry, and fishing; Mining and quarrying; Food, beverages and tobacco; Textiles, leather and footwear; Basic metals and fabricated metal; Machinery NEC; Electrical and optical equipment; Manufacturing NEC. Industries that are classified as non-tradable (NT) include: Electricity, gas, and water supply; Construction; Wholesale trade; Retail trade ex. motor vechicles; Hotels and restaurants; Transport and storage; Health and social work. Countries includes in the sample: AUS, AUT, BEL, CYP, DNK, ESP, FIN, FRA, GRC, IRL, ITA, JPN, KOR, NLD, PRT, SWE, and USA.

6 Results

In this section I present the results from simulating the calibrated model. The first subsection examines the implied profile of growth as the home country catches up to the foreign country and the productivity frontier. And the second subsection examines the compositional evolution of the home country over time.

6.1 Catch up growth profile

Figure 5 plots the model's implied average annual growth rate between 1960 to 2011 for countries that has an initial GDP per capita ranging from \$2,000 to \$20,000 in 1960 (in 1995 PPP units). The results suggest that the model is able to replicate the kind of growth rates seen in the data for the preferred values of $\mu_{NT} = \mu_{TR} = 0.01$.

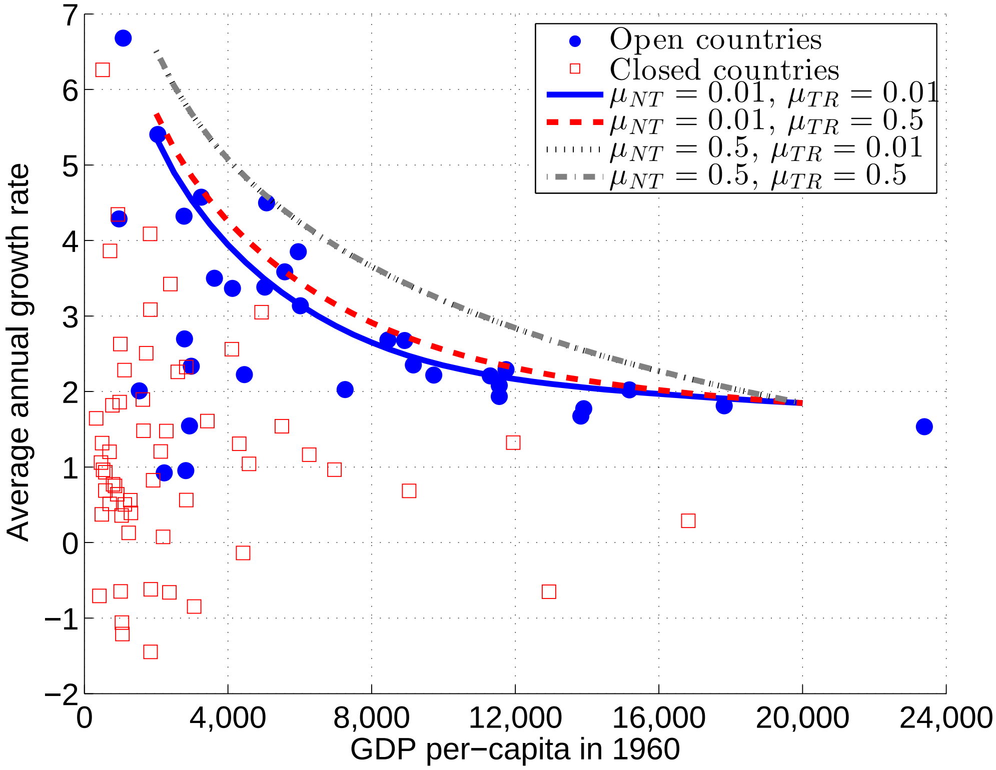
As a robustness check, figure 5 also shows the implied growth rates when we allow the values of μ_{NT} and μ_{TR} to be higher. Raising the value of μ_{NT} from one percent to 50 percent shifts the growth profile from the solid blue line to the black dotted line (which lies very close to the gray dash-dot line). Raising the value of μ_{TR} on the other hand has very little impact on the growth profile. This is because gains in productivity in the tradable sector work only through the channel of raising export receipts, where as increases in non-tradable productivity affect directly the market for goods in the home country.

One could improve the fit of the model to the data by fine tuning the choice of μ_{NT} and μ_{TR} further. In addition, the parametrization of the model has enough flexibility that μ_{NT} and μ_{TR} could be allowed to differ in such a way that the model still provides a good fit to the data. I do not do so as the simple case considered here is sufficient to illustrate the ability of the model to match the data.

6.2 Home country dynamics

In this subsection, I show the evolution of the key variables within the home country over time. For the simulations, I focus on the particular case where the initial GDP per capita in the home country is equal to 2,000. This is similar to the income levels of Taiwan and

Figure 5: Model implied growth rate for various μ_{NT} and μ_{TR} values



The model implied growth rate curve is found by simulating the model from 1960 to 2011 and averaging the annual growth rate in GDP for various starting levels of GDP per capita. Each line is found for different values of μ_{NT} and μ_{TR} (the fraction of meetings non-tradable and tradable firms in the home country have with foreign firms).

Indonesia in 1960. I use the calibration such that $\mu_{NT} = \mu_{TR} = 0.01$ for all the results in this simulation.

Figure 6 shows the weighted average level of productivity in the home country's tradable (exporters) and non-tradable sectors relative to the average level of productivity in the foreign country. Initially the average level of productivity in the home country's sectors is around 10 percent of the average level of productivity in the frontier (close to the 10 percent difference in initial GDP per capita). The non-tradable sector has slightly higher initial productivity than the tradable sector to explain why the initial share of imports to GDP is below that in the steady state (ω). Low tradable productivity in the home country's exporters results in less revenue from export receipts. And by the balanced trade assumption, low export receipts results in low levels of imports into the home country.

The productivity gap between the two sectors in the home country rapidly disappears as both sectors benefit from the inflow of productivity ideas from the foreign country.[13] And after around 30 years, the average level of productivity in the home country is around half the level of productivity in the foreign country.

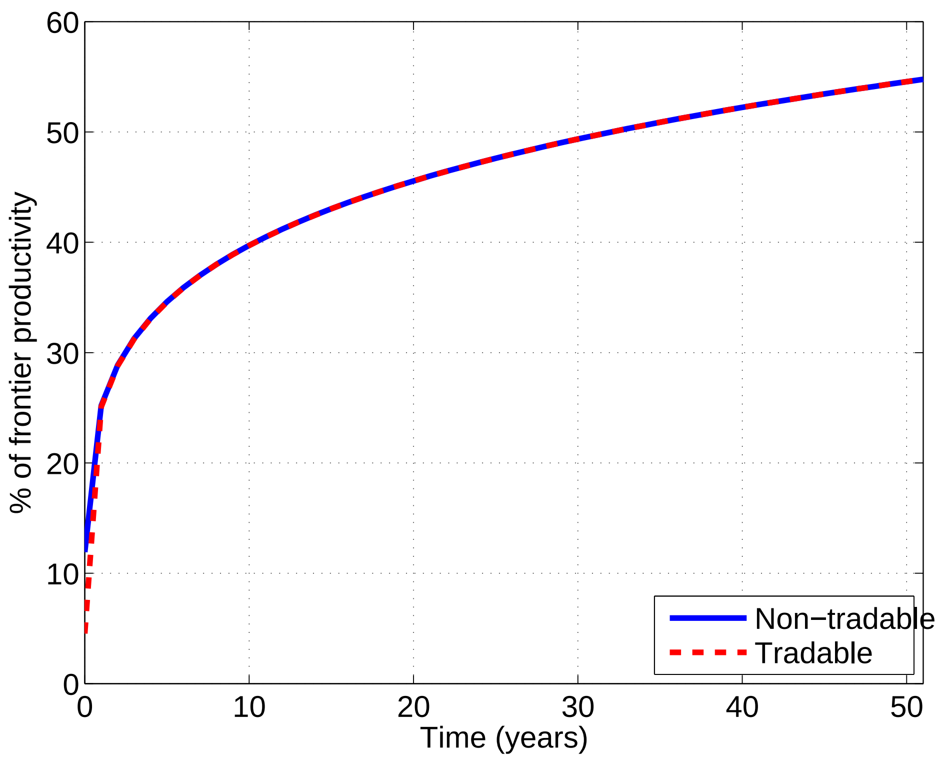
Figure 7 plots the annual growth rate (subfigure 7a) and the log-level (subfigure 7b) of consumption in both countries over time. The non-tradable sector shows higher growth rates than the tradable sector for the home country due to the substitution effect working in the favor of non-tradable consumption. Productivity growth in the non-tradable sector is higher than productivity growth in the foreign country (who produce all the tradable goods consumed by the home country). This results in the relative price of of non-tradables to tradables declining, creating a substitution effect towards non-tradable consumption.

6.3 Contribution of foreign productivity to growth in the home country

Estimates by Eaton and Kortum (1996) using a quality ladders model suggest that the majority of productivity growth in most OCED countries is sourced from other countries.

[13] Studies at the firm level suggest that firms in the export sector are more productive than other firms (see Wagner (2007) for a survey of the literature). In this model framework, $\mu_{TR} > \mu_{NT}$ would result in the tradable sector (the exporters) converging to the productivity frontier faster than the non-tradable firms. One could easily increase the value of μ_{TR} and lower the value of μ_{NT} to match this fact without significantly impacting upon the catch up growth rates shown in the previous subsection.

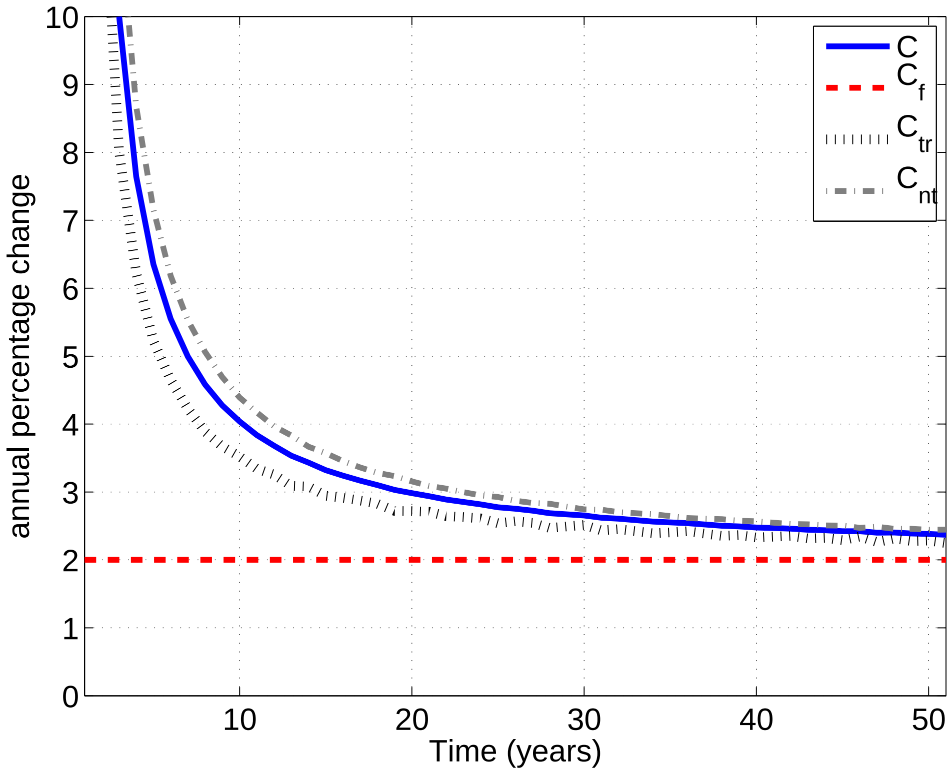
Figure 6: Average level of productivity in each sector relative to foreign productivity



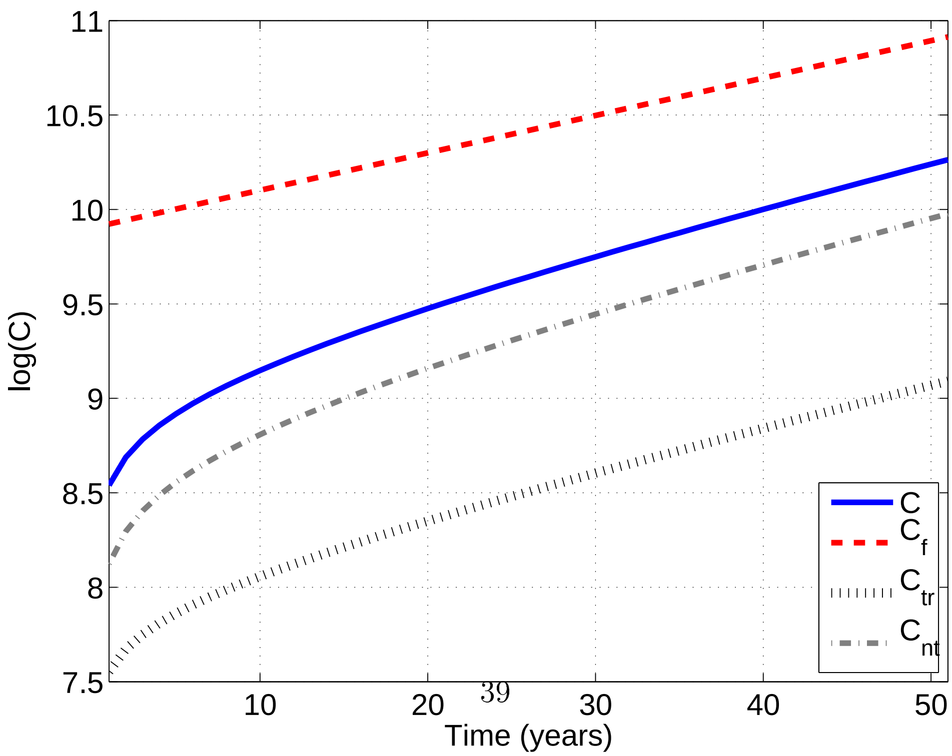
Each line denotes the average level of productivity in the home country's tradable and non-tradable firms ($\int z f_i(z,t)dz$) as a percentage of the average level of productivity in the foreign country at time t. The initial difference in average productivity between home country tradable and non-tradable firms rapidly disappears, and in the limit the distribution of productivity in the home country converges to match that in the foreign country.

Figure 7: Consumption paths for a simulated country

(a) Consumption growth



(b) Consumption levels



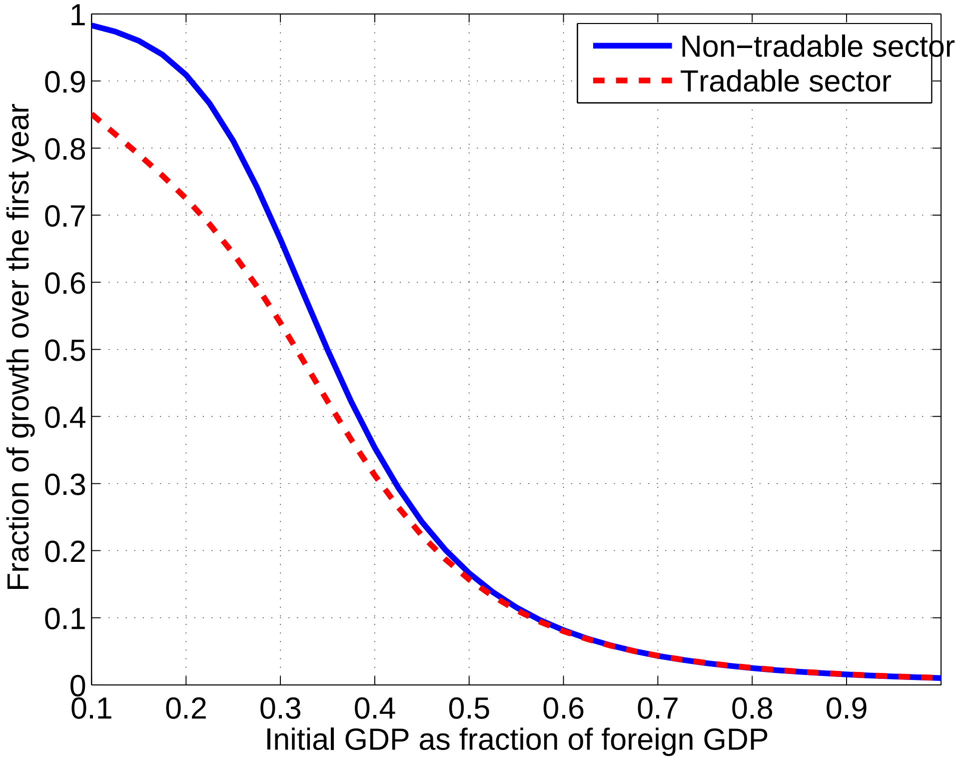
Smaller countries such as Greece, Ireland, and Portugal receive over 99 percent of their productivity growth from other countries. Most other larger OECD countries receive over 90 percent of their productivity growth from overseas, apart from US, Japan, France and Germany, all who are the R&D intensive countries. In this subsection, I examine the implied contribution to productivity growth in the home country from the foreign country (overseas) to see how it compares to the estimates of Eaton and Kortum (1996).

The meeting rates between the home country firms and the foreign firms (μ_{NT} and μ_{TR}) are not sufficient to fully describe the influence that the foreign country has on the growth in productivity in the home country. The influence of the foreign country also depends on the difference in scale of the productivity distributions in the two countries. Because the distribution of productivity in the foreign country is more advanced than the home country's productivity distribution (it first order stochastically dominates it), there are two main channels through which the foreign productivity benefits the home country beyond simply increasing the number of meetings per period. First, conditional upon meeting a firm, the home country is much more likely to meet a foreign firm who has a higher productivity level than itself than it is to meet a home country firm who has higher productivity. Therefore, meetings with foreign firms will, on average, lead to more increases in productivity for home country firms. And second, conditional upon meeting a firm with higher productivity, meeting a foreign firm yields a larger increase in productivity than meeting a home country firm, on average, given that the distribution of foreign productivity first order stochastically dominates the home country's productivity distribution.

Using the differential equations that govern the evolution of productivity in each sector of the model in this paper, it is possible to decompose the change in average productivity in the home country's two sectors into the contribution from the foreign country and the contribution from the home country. However, because domestic firms are able to learn indirectly foreign productivity ideas, by learning from other domestic firms who had previously learned from foreign firms, the contribution of the foreign country to productivity growth in the home country is time dependent.

For simplicity, I focus the examination on the contribution to growth over the first year of the model ($t = 0$ to $t = 1$). Given the distributions of productivity $F_{TR}(z,t)$ and $F_{NT}(z,t)$, I compute the average productivity level in both the tradable and non-tradable sectors of the home country at time $t = 0$ and $t = 1$ to calculate the growth rate in average productivity in each sector over the first year. I then calculate the counter-factual growth rate in average

Figure 8: Fraction of growth in productivity due to the foreign country



Each line denotes the faction of the growth in the average level of productivity during the first year in the home country's tradable and non-tradable firms ($\left(\int z^{1/(\nu-1)} f_i(z,t)dz\right)^{\nu-1}$) *due to learning from foreign firms. The remaining fraction represents the growth in the average level of productivity that is due to learning for the home country's domestic (non-tradable) firms.*

productivity when the home country does not learn from the foreign country. I.e. the meeting rate with the foreign country is zero, $\beta = \gamma = 0$.

The difference between these growth rate and the counter-factual growth rate is the contribution of the foreign country to productivity growth in the home country. This calculation is repeated for each initial GDP per capita level (between 2,000 and 20,000) and the results are plotted in figure 8.

It is important to note here that this calculation for the contribution of the foreign country to productivity growth in the home country sector attributes the initial distribution of productivity in the home country at time $t = 0$ entirely to the home country. Because period $t = 0$ in the models calibration refers to 1960, this calculation implicitly underestimates the diffusion of productivity from the foreign country by ignoring productivity diffusion flows that occurred before 1960. Therefore, these estimates should be viewed as a lower bound on the true contribution of the foreign country.

According to the results shown in figure 8, For the poorest countries the foreign country contributes the vast majority of growth. The foreign country contributes nearly all of the growth in average productivity in the non-tradable sector and around 85 percent of the growth in the tradable sector. The contribution of the foreign country to growth is lower in the tradable sector, because the even when the home country's tradable firms are meeting only non-tradable, the distribution of non-tradable productivity first order stochastically dominates the distribution of tradable productivity. Therefore, an average tradable firm received more benefit on average from meeting non-tradable firms than does a non-tradable firm meeting the same pool of non-tradable firms.

The contribution of the foreign country to the home country's productivity growth rapidly declines as the initial level of GDP per capita in the home country increase. And when the home country starts with a level of GDP per capita very close to that of the foreign country, the fraction of growth resulting from the foreign country is very low, around one percent. This is due to the home country having productivity distributions that are very similar in scale to the foreign country (and hence the similar GDP per capita between countries). Therefore, meeting a firm from the foreign country does not result in significantly more productivity gain on average when compared to meeting a firm from the home country. And the main contribution of the foreign country to the home country's growth is through the increase in number of meetings when compared to the case of only meeting home country firms. And with the values of $\mu_{TR} = \mu_{NT} = 0.01$, this effect only adds around one percent to the growth of average productivity in the home country.

The estimated contribution of the foreign country to productivity growth in this model during the first period are significantly lower than the estimates of Eaton and Kortum (1996) for all but the poorest countries. However, the dynamic dimension of the model in this paper means that the contribution of the foreign country to productivity growth in the home home rises over time as more meetings with foreign firms occur, and foreign ideas already adopted by the home country firm's are diffused further among home country firms.

7 Optimal tariffs

The small open economy model developed in section 3 and 4 is a special limiting case of the two-country framework. While this modeling approach is convenient in terms of developing a model whereby the productivity frontier is exogenous, it does restrict the model's ability to address policy related questions such as the optimal tariff from the point of view of the home country. In this section, I relax some of the modeling assumptions made previously to address this specific question about tariffs.

Imposing tariffs on imports affects a home country through two main channels. First there is the intra-temperal channel where by increasing the price of imports, local producers become more competitive, and gain a larger market share of the tradable sector. This increases the demand for labor in the home country and raises GDP. And second, there is an inter-temporal channel where replacing some of the productive overseas producers with less productive home producers, skews the distribution of productivity of firms operating in the home country's tradable market downwards. And this will adversely affect the ability of all firms in the home country to meet and learn from other firms, thereby lowering the rate of productivity diffusion in the home country.

Ex-ante, it is not clear which of these two channels will dominate. The answer will be dependent upon both the size of the country, and how far behind the productivity frontier it is. If the home country is relatively large, a gain in the market share for home country producers will lead to a large increase in output for the home country. And if the home country is relatively close to the productivity frontier, then replacing productive foreign firms with slightly less productive domestic firms will not dramatically impact upon the growth rate of productivity in the home country.

In subsection 7.1, I extend the model to include the two channels previously mentioned through which tariffs can influence the home country discussed above. And in subsection 7.2 I present the results for simulating a sample economy for various tariff rates to see how GDP growth is affected by the level of the tariff.

7.1 Extension to the model

Appendix C presents the extended model equations in their entirety. Here I present and discuss the main changes to the model structure, and assumptions made. The main modification made to the model is to now introduce some "size" for the home country where by it is able to obtain some market share of its own tradable sector.[14]

While the home country now has enough size to hold some market share of its own tradable section, I assume that the home country is not so large as to affect the foreign country/productivity frontier. This is a departure from the two-country, two-sector framework (where in the home country has some market share in the foreign country's tradable sector), but retains the ability to take the productivity frontier as exogenous for the story of catch up growth. In addition, I assume that the foreign country (representing the rest of the world) does not impose any tariff on the home country in response to its tariff policies, given the small size of the home country.

Static trade model

The home country's government imposes a tariff of τ on price of each individual tradable good imported into the home country. Therefore the price of an individual tradable good produced in the foreign country and consumed in the home country is given by $(1+\tau)w_F/(z_F\kappa_{H,F})$, where recall from before that w_F is the wage rate in the foreign country, and $\kappa_{H,F}$ is the iceberg trade cost of shipping one unit of good z_F from the foreign country to the home country.

The aggregate price of tradable goods in the home country is now a sum of two components

$$P_{TR}^{1-\nu} = \int_{B_{F,H}} \left(\frac{(1+\tau)w_F}{\kappa_{H,F}z_F}\right)^{1-\nu} f_F(z_F,t)dz_F + \int_{B_{H,H}} \left(\frac{w}{z}\right)^{1-\nu} f(z,t)dz$$

where $B_{F,H}$ is the subset of $z \in \mathbb{R}_+$ such that the foreign country is the least cost supplier of in the home country, and $B_{H,H}$ is the subset of productivity/goods that the home country tradable firms are the least cost supplier of. Alternatively, the expression for the price of the

[14] Recall that in the limiting case of the SOE model, the home country's share of its own tradable sector is insignificant, and can thus be ignored.

aggregate tradable good in the home country can be expressed as

$$\begin{aligned} P_{TR}^{1-\nu} &= \int_{\mathbb{R}_+} \left(\frac{(1+\tau)w_F}{\kappa_{H,F} z_F}\right)^{1-\nu} f_F(z_F,t) F\left(\frac{w\kappa_F}{(1+\tau)w_F} z_F, t\right) dz_F \\ &\quad + \int_{\mathbb{R}_+} \left(\frac{w}{z}\right)^{1-\nu} f(z,t) F_F\left(\frac{(1+\tau)w_F}{w\kappa_F} z, t\right) dz \end{aligned}$$

From this expression we can see that the value of τ has two effects on the price of tradables in the home country. First, it directly increases the price of the individual tradable goods the foreign country supplies ($(1+\tau)w_F/(\kappa_{H,F}z_F)$). And second, it has a selection effect (through the CDF terms F and F_F) that shifts the marginal goods away from foreign producers to domestic producers.

And given that home country tradable firms now produce goods for the home country market, the expression for excess labor demand in the home country ($E_H(w,w_F)$) now becomes

$$\begin{aligned} E_H(w,w_F) &= L_F w_F (1-\omega) \left(\frac{P_{TR}^F}{P^F}\right)^{1-\eta} D_{F,H} \\ &\quad + L_H w \left[(1-\omega)\left(\frac{P_{TR}}{P}\right)^{1-\eta} D_{H,H} + \omega \left(\frac{P_{NT}}{P}\right)^{1-\eta}\right] - L_H w \end{aligned}$$

where $D_{H,H}$ represents the home country's market share of the home country's tradable sector and is given by the expression

$$D_{H,H} = \int_0^\infty F_{TR}^F\left(\frac{w_F}{\kappa_{H,F} p}\right) d\left[1 - F_{TR}\left(\frac{w}{p}\right)\right]$$

and L_H is the measure of the labour force in the home country.

The revenues raised by the tariff on imports is repatriated as a lump sum to the households in the home country. And the household budget constraint becomes

$$PC = w + TR$$

where TR is the amount of transfers, expressed as

$$TR = \int_{B_{H,F}} \tau \frac{w_F}{\kappa_{H,F} z_F} c_{TR}(z_F) f_F(z_F) dz_F$$

Growth model

As shown above, the introduction of the tariff changes the composition of firms who operate in the home country's tradable sector. A higher tariff crowds out productive foreign firms, and replaces them with less productive home country firms. As a result, both home country tradable and non-tradable firms have the chance to meet home country tradable firms via their meetings with the firms who operate in the home country tradable market. Let $G(z,t)$ denote the CDF of the distribution of productivity for firms operating in the home country's tradable market at time t. The expression for this distribution is given by

$$G(z,t) = \int_0^z f_{TR}(y,t) F_{TR}^F\left(\frac{(1+\tau)w_F}{w_H \kappa_{H,F}} y\right) dy + \int_0^z f_{TR}^F(y,t) F_{TR}\left(\frac{w_H \kappa_{H,F}}{(1+\tau)w_F} y\right) dy$$

The differential equations for updating the distribution of productivity of non-tradable and tradable firms in the home country becomes

$$\frac{\partial \log[F_{NT}(z,t)]}{\partial t} = \beta \log[G(z,t)] + \rho \log[F_{NT}(z,t)] \tag{20}$$

$$\frac{\partial \log[F_{TR}(z,t)]}{\partial t} = \gamma \log[G(z,t)] + \delta \log[F_{NT}(z,t)] \tag{21}$$

where $G(z,t)$ has now replaced $F_F(z,t)$ (the distribution of foreign productivity) in the two equations above.

7.2 Results for optimal tariff rates

Having included tariffs into the structure of the model now allows us to examine how the level of tariffs imposed by the home country impacts upon the growth of GDP.

To analyze the level of optimal tariffs, I simulate the model for an artificial home country, who begins with a initial level of GDP per capita of 10 percent of the rest of the world/productivity frontier (the same as in the previous section). The size of the home country labor force (L_H) is calibrated to be one percent of the total world population. This makes the country similar in size to modern day England, Germany, or Turkey. I simulate the model forward 51 years (1960 to 2011) for various tariff rates and compare the level of GDP in 2011 to that in the scenario of zero tariffs. The results are plotted in figure 9 in terms of the annual average percentage gain in GDP.[15]

Figure 9 shows that for a relatively large range of tariffs (zero percent to 90 percent of the price of imports), tariffs make the home country better off. The additional demand for home country tradable goods outweighs the inter-temperal channel of having a worse distribution of firms to meet ($F_F(z,t)$ first order stochastically dominated $G(z,t)$). Two factors help contribute to this result. First, raising the tariff eliminates the least productive foreign firms first. The most productive foreign firms are still able to operate in the home country market, and hence home country firms are still able to meet the most productive foreign firms who yield the greatest productivity boost. The second factor is that starting from a productivity level far below the frontier means that even relatively large tariffs have a small selection effect on the more advanced foreign firms. Foreign firms are initially so much more productive than the home country firms, the vast majority of them are still able to overcome relatively small tariffs.

The optimal tariff rate is figure 9 is around 30 percent for this simulated country. This optimal tariff results in GDP for the home country that is on the order of eight percent higher in 2011 than in the scenario where the home country had zero tariffs.

8 Conclusion

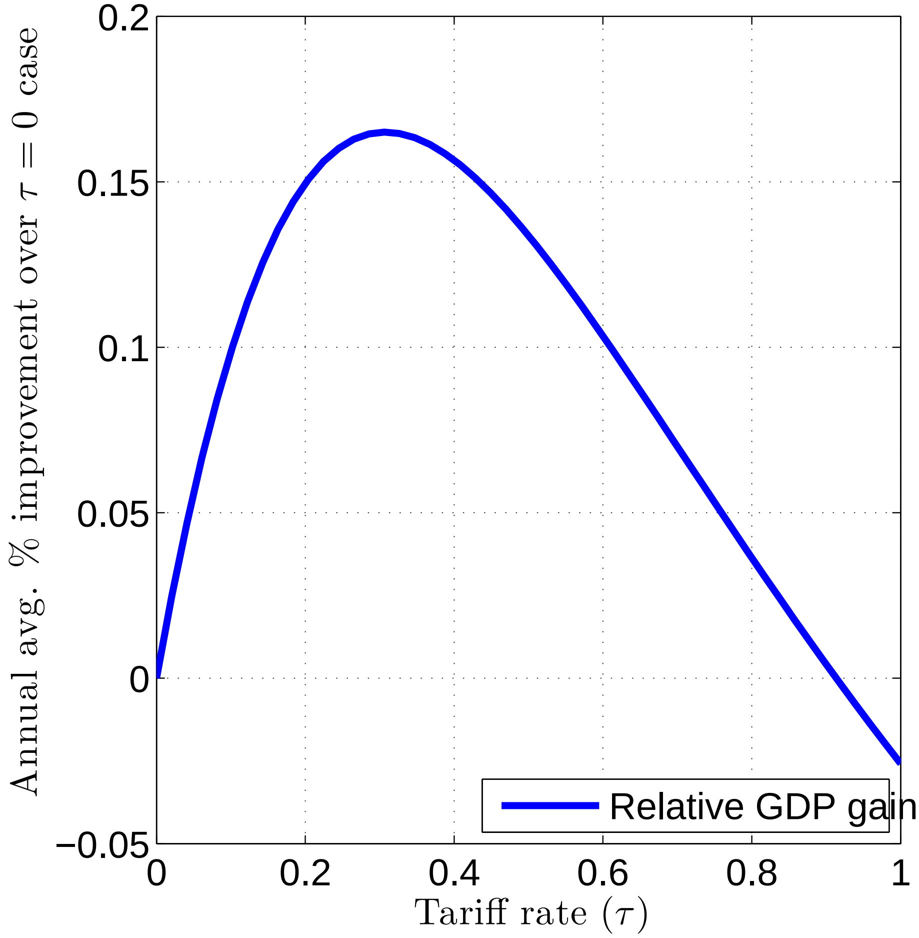
This paper develops a model of catch-up growth in a small open economy through the process of a diffusion of productivity from the productivity frontier. The model is shown the be able to replicate the profile of catch up growth in both GDP and average level of productivity seen in open economies since 1960 when home country firms have a relatively

[15]Even in the case when $\tau = 0$, the modified model where the home country has "size" in its own tradable market will not produce equivalent results as the small open economy case discussed previous. The small open economy model is a limiting case for when $L_H \to 0$. Therefore, for a relatively small home country, the results will be similar to the SOE case.

In addition, because of $G(z,t)$, the distributions of $F_{NT}(z,t)$ and $F_{TR}(z,t)$ cannot be expressed easily as functions of the CDFs at time zero (they are now dependent upon the historical path of wages from the static trade model). Therefore, I discretize both the time dimension and the support for z in the model when simulating it, which may introduce some small numerical inaccuracies.

To give a sense for the scale of difference between the SOE model results from the previous section, and the zero tariff results for the home country of size one percent in this section, At the end of the simulated time period (2011, 51 years), the log of GDP in the SOE model is equivalent to 0.9404 of the log of foreign GDP, while in the case of the zero tariff model, the log of GDP is equivalent to 0.9583 of the log of foreign GDP.

Figure 9: GDP gain relative to free-trade baseline



The annual average percentage improvement over the scenario where $\tau = 0$, is computed based on simulating the model 51 years (1960 to 2011), and then comparing the level of GDP at the end of the simulation relative to the level of GDP at the end of the simulation when $\tau = 0$.

low, on the order of one percent, fraction of their meeting and learning opportunities with foreign firms. Simulating an extended version of the model that incorporates import tariffs for the home country reveals that for a small, less developed country the gains in domestic demand for home produced tradable goods outweighs the loss in productivity diffusion caused be restricted more productive firms from operating in the home market. Thus import tariffs are optimal for small developing countries, yet the net benefit they bring is relatively muted.

A Derivation of the excess labor demand function

In this appendix I derive the excess labor demand function for the home country. I begin with the case where both countries are of similar magnitude, and then consider the limiting case of a small open economy where the relative size of the home country goes to zero.

A.1 Excess labor demand when both countries are of a similar magnitude

Balanced trade for the home country implies that the export receipts from tradable firms selling to the foreign market are equal to the amount spent on imports. In the home country, individual consumers spend a total of $P_{TR}C_{TR}$ on tradable goods. Let $D_{H,F}$ denote the foreign country's share of the tradable market. Therefore, the total outflows from the home country is given by

$$LP_{TR}C_{TR}D_{H,F}$$

where L is the measure of households in the home country.

The total inflows into the home country are sourced from the home country's share of foreign spending on tradable goods measured as

$$L^F P^F_{TR}C^F_{TR}D_{F,H}$$

where L_F is the measure of households in the foreign country, P^F_{TR} is the price of tradable goods in the foreign country, C^F_{TR} is the consumption of tradable goods in the foreign country, and $D_{F,H}$ is the home country's market share in the foreign market.

Therefore balanced trade requires

$$LP_{TR}C_{TR}D_{H,F} \quad = \quad L^F P^F_{TR}C^F_{TR}D_{F,H}$$

Each dollar spent on non-tradable goods in the home country, and each dollar spent on domestically produced tradable goods is also an inflow into the home country. Adding these two flows to both sides of the trade balance equation and simplifying yields

$$L(P_{TR}C_{TR} + P_{NT}C_{NT}) = L^F P_{TR}^F C_{TR}^F D_{F,H} + LP_{TR}C_{TR}D_{H,H} + LP_{NT}C_{NT} \tag{22}$$

The equation above states that total expenditure by the home country (the left hand side) is financed from three sources of income (the right hand side): income for exports to the foreign market, income from domestically produced and consumed tradables, and income from non-tradables.

Using the home country's budget constraint (equation 2), optimal choices of C_{NT}, C_{TR} and (equations 3 and 4 respectively), as well as the corresponding optimal choice of C_{TR}^F in the foreign country, equation 22 can be rewritten as

$$Lw = L^F(1-\omega)\left(\frac{P_{TR}^F}{P^F}\right)^{1-\eta} D_{F,H} + Lw\left[(1-\omega)\left(\frac{P_{TR}}{P}\right)^{1-\eta} D_{H,H} + \omega\left(\frac{P_{NT}}{P}\right)^{1-\eta}\right]$$

We can view this equation as an excess labor demand function and define $E_H(w, w_F)$ as

$$\begin{aligned} E_H(w, w_F) &= L^F(1-\omega)\left(\frac{P_{TR}^F}{P^F}\right)^{1-\eta} D_{F,H} \\ &\quad + Lw\left[(1-\omega)\left(\frac{P_{TR}}{P}\right)^{1-\eta} D_{H,H} + \omega\left(\frac{P_{NT}}{P}\right)^{1-\eta} - 1\right] \end{aligned} \tag{23}$$

The home country's share of the foreign market ($D_{F,H}$) represents the probability that the home country is the least cost provider in the foreign market across all prices

$$D_{F,H} = \int_0^\infty [1 - R_{F,F}(p)] dR_{F,H}(p)$$

where $R_{F,F}$is the probability that the foreign country can supply good z to the foreign market at a price less than p, and $R_{F,H}(p)$ is the probability that the home country can supply good z to the foreign market at a price less than p. Using the distributions of productivity in each country and the expressions of the price of good z produced by each country, $D_{F,H}$ can also be expressed as

$$D_{F,H} = \int_0^\infty F_F\left(\frac{w_F}{p}\right) d\left[1 - F_{TR}\left(\frac{w}{\kappa_{F,H}p}\right)\right]$$

Likewise, the home country's share of its own tradable market can be expressed as

$$\begin{aligned} D_{H,H} &= \int_0^\infty \left[1 - R_{H,F}(p)\right] dR_{H,H}(p) \\ &= \int_0^\infty F_F\left(\frac{w_F}{\kappa_{H,F}p}\right) d\left[1 - F_{TR}\left(\frac{w}{p}\right)\right] \end{aligned}$$

A.2 Excess labor demand in the limiting case of a small open economy

Next I consider the limiting case of the home country being a small open economy where its relative size goes to zero ($L \to 0$). Begin by considering diving the home country of size L up into n identical and independent countries of size $\hat{L}$, that each only trade with the foreign country (and not each other). Therefore

$$\begin{aligned} L &= n\hat{L} \\ F_{TR}(z,t) &= \left[\hat{F}_{TR}(z,t)\right]^n \\ F_{NT}(z,t) &= \left[\hat{F}_{NT}(z,t)\right]^n \end{aligned}$$

Considering the limiting case where the home country has insignificant size is equivalent to dividing the large home country up into an infinite number of smaller countries ($n \to \infty$) whose size is infinitely small $\hat{L} \to 0$, and focusing on the case of one of these particular countries.

Let us consider the excess labor demand equation for the economy of size $\hat{L}$, denoted $\hat{E}_H(w, w_F)$. The limit of the excess demand equation as $\hat{L} \to 0$ (or equivalently $n \to \infty$) is the same as the limit of $\hat{E}_H(w, w_F)/\hat{L}$ for $\hat{L} > 0$.

$$\begin{aligned} \hat{E}_H(w, w_F) &= w^F L^F (1-\omega)\left(\frac{P^F_{TR}}{P^F}\right)^{1-\eta} \hat{D}_{F,H} + \hat{L}w\left[(1-\omega)\left(\frac{P_{TR}}{P}\right)^{1-\eta}\hat{D}_{H,H} + \omega\left(\frac{P_{NT}}{P}\right)^{1-\eta} - 1\right] \\ \frac{\hat{E}_H(w, w_F)}{\hat{L}} &= w^F L^F (1-\omega)\left(\frac{P^F_{TR}}{P^F}\right)^{1-\eta} \frac{\hat{D}_{F,H}}{\hat{L}} + w\left[(1-\omega)\left(\frac{P_{TR}}{P}\right)^{1-\eta}\hat{D}_{H,H} + \omega\left(\frac{P_{NT}}{P}\right)^{1-\eta} - 1\right] \end{aligned}$$

where

$$\begin{aligned}
\hat{D}_{F,H} &= \int_0^\infty F_F\left(\frac{w_F}{\kappa_{F,F}p}\right) d\left[1-\hat{F}_{TR}\left(\frac{w}{\kappa_{F,H}p}\right)\right] \\
\hat{D}_{H,H} &= \int_0^\infty F_F\left(\frac{w_F}{\kappa_{H,F}p}\right) d\left[1-\hat{F}_{TR}\left(\frac{w}{p}\right)\right]
\end{aligned}$$

Note that I assume that the foreign country has an iceberg trade cost of supplying tradable goods to itself. Therefore, we can think of the foreign country as being a collection of smaller identical countries. In addition, it allows the foreign country to represent the limit of the home country. For simplicity, I assume $\kappa_{F,F} = \kappa_{H,F}$.

Taking the limit of the excess demand function requires examining $\lim_{\hat{L}\to 0} \hat{D}_{F,H}/\hat{L}$ and $\lim_{\hat{L}\to 0} \hat{D}_{H,H}$. Rewriting these expressions in terms of $F_{TR}(\cdot)$ instead of $\hat{F}_{TR}(\cdot)$ and taking the limit as $n \to \infty$

$$\begin{aligned}
\frac{\hat{D}_{F,H}}{\hat{L}} &= \hat{L}^{-1}\int_0^\infty F_F\left(\frac{w_F}{\kappa_{F,F}p}\right) d\left[1-F_{TR}\left(\frac{w}{\kappa_{F,H}p}\right)^{1/n}\right] \\
&= \frac{n}{L}\int_0^\infty F_F\left(\frac{w_F}{\kappa_{F,F}p}\right)\frac{1}{n}\frac{w}{\kappa_{F,H}p^2} f_{TR}\left(\frac{w}{\kappa_{F,H}p}\right) F_{TR}\left(\frac{w}{\kappa_{F,H}p}\right)^{1/n-1} dp \\
&= L^{-1}\int_0^\infty F_F\left(\frac{w_F}{\kappa_{F,F}p}\right)\frac{w}{\kappa_{F,H}p^2} f_{TR}\left(\frac{w}{\kappa_{F,H}p}\right) F_{TR}\left(\frac{w}{\kappa_{F,H}p}\right)^{1/n-1} dp \\
\lim_{\hat{L}\to 0}\frac{\hat{D}_{F,H}}{\hat{L}} = \lim_{n\to\infty}\frac{\hat{D}_{F,H}}{\hat{L}} &= \lim_{n\to\infty} L^{-1}\int_0^\infty F_F\left(\frac{w_F}{\kappa_{F,F}p}\right)\frac{w}{\kappa_{F,H}p^2} f_{TR}\left(\frac{w}{\kappa_{F,H}p}\right) F_{TR}\left(\frac{w}{\kappa_{F,H}p}\right)^{1/n-1} dp \\
&= L^{-1}\int_0^\infty \frac{w}{\kappa_{F,H}p^2} f_{TR}\left(\frac{w}{\kappa_{F,H}p}\right) F_F\left(\frac{w_F}{\kappa_{F,F}p}\right) F_{TR}\left(\frac{w}{\kappa_{F,H}p}\right)^{-1} dp
\end{aligned}$$

and

$$\begin{aligned}
\hat{D}_{HH} &= \int_0^\infty F_{F,t}\left(\frac{w_F}{\kappa_{H,F}p}, t\right)\frac{1}{n}\frac{w}{p^2} f_{TR}\left(\frac{w}{p}, t\right)\left[F_{TR}\left(\frac{w}{p}, t\right)\right]^{1/n-1} dp \\
\lim_{n\to\infty}\hat{D}_{H,H} &= 0
\end{aligned}$$

Therefore

$$\lim_{\hat{L}\to 0}\frac{\hat{E}_H(w, w_F)}{\hat{L}} = w^F L^F(1-\omega)\left(\frac{P^F_{TR}}{P^F}\right)^{1-\eta}\lim_{n\to\infty}\left(\frac{\hat{D}_{F,H}}{\hat{L}}\right) + w\left[+\omega\left(\frac{P_{NT}}{P}\right)^{1-\eta} - 1\right]$$

B Complete small open economy model – static trade model equations

This appendix presents the complete static trade model for the small open economy case

B.1 Home country

Home country's budget constraint:

$$PC = P_{NT}C_{NT} + P_{TR}C_{TR} \quad = \quad w$$

The optimal choices of C_{NT} and C_{TR}:

$$\begin{aligned} C_{NT} &= \omega \left(\frac{P_{NT}}{P}\right)^{-\eta} C \\ C_{TR} &= (1-\omega) \left(\frac{P_{TR}}{P}\right)^{-\eta} C \end{aligned}$$

P is the ideal price index:

$$P \quad = \quad \left[\omega P_{NT}^{1-\eta} + (1-\omega) P_{TR}^{1-\eta}\right]^{1/(1-\eta)}$$

The price of the non-tradable composite:

$$P_{NT} \quad = \quad w \left[\int_{\mathbb{R}_+} z^{\sigma-1} f_{NT}(z,t) d(z)\right]^{1/(1-\sigma)}$$

The price of the tradable composite:

$$P_{TR} \quad = \quad \frac{w_F}{\kappa_{H,F}} \left[\int_{\mathbb{R}_+} z^{\nu-1} f_F(z,t) dz\right]^{1/(1-\nu)}$$

Excess labor demand in the home country

$$E_H(w, w_F)/\tilde{L} \;=\; w_F(1-\omega)\left(\frac{P^F_{TR}}{P^F}\right)^{1-\eta}\hat{D}_{F,H,t} + w\left[\omega\left(\frac{P_{NT}}{P}\right)^{1-\eta} - 1\right]$$

where

$$\hat{D}_{F,H,t} \;=\; \frac{L_F}{\tilde{L}}\lim_{n\to\infty}\int_0^\infty \frac{w}{\kappa_{F,H}p^2} f_{TR}\left(\frac{w}{\kappa_{F,H}p}, t\right) F_{TR}\left(\frac{w}{\kappa_{F,H}p}, t\right)^{-1} F_F\left(\frac{w_F}{p}, t\right) dp$$

B.2 Foreign country

Foreign country's budget constraint:

$$P^F C^F = P^F_{NT}C^F_{NT} + P^F_{TR}C^F_{TR} \;=\; w_F$$

The optimal choices of C_{NT} and C_{TR}:

$$\begin{aligned} C^F_{NT} &= \omega\left(\frac{P^F_{NT}}{P^F}\right)^{-\eta} C^F \\ C^F_{TR} &= (1-\omega)\left(\frac{P^F_{TR}}{P^F}\right)^{-\eta} C^F \end{aligned}$$

P^F is the ideal price index:

$$P^F \;=\; \left[\omega\left(P^F_{NT}\right)^{1-\eta} + (1-\omega)\left(P_{TR}\right)^{1-\eta}\right]^{1/(1-\eta)}$$

The price of the non-tradable composite:

$$P^F_{NT} \;=\; w_F\left[\int_{\mathbb{R}_+} z^{\sigma-1} f_F(z,t)d(z)\right]^{1/(1-\sigma)}$$

The price of the tradable composite:

$$P^F_{TR} \;=\; \frac{w_F}{\kappa_{H,F}}\left[\int_{\mathbb{R}_+} z^{\nu-1} f_F(z,t)dz\right]^{1/(1-\nu)}$$

And the foreign wage rate is normalized to one, clearing the labor market.

C Extended model for tariff application

The key equations for the home country section of the extended model that incorporates tariffs are presented below.

C.1 Static trade model

The home country budget constraint is given by:

$$PC = w + TR$$

where P is the aggregate price index, C is the aggregate consumption good (GDP), w is the wage rate in the home country, and TR is the lump-sum transfer of tariff revenues back to the house each period.

Lump-sum transfer:

$$\begin{aligned} TR &= \int_{B_{H,F}} \tau p_{TR}^{F}(z_F) c_{TR}(z_F) f_F(z_F) dz_F \\ &= \int \frac{\tau w_F}{\kappa_{HF} z_F} c_{TR}(z_F) f_F(z_F) F_{TR}\left(\frac{w\kappa_F}{(1+\tau)w_F} z_F\right) dz_F \\ &= \frac{\tau w_F}{\kappa_{HF}} P_{TR}^{\nu} C_{TR} \left(\frac{(1+\tau)w_F}{\kappa_{HF}}\right)^{-\nu} \int z_F^{\nu-1} f_F(z_F) F_{TR}\left(\frac{w\kappa_F}{(1+\tau)w_F} z_F\right) dz_F \end{aligned}$$

Aggregate price index:

$$P = \left[\omega P_{NT}^{1-\eta} + (1-\omega) P_{TR}^{1-\eta}\right]^{1/(1-\eta)}$$

Price of the composite non-tradable good (P_{NT}):

$$P_{NT} = w\left[\int_{\mathbb{R}_+} z^{\sigma-1} f_{NT}(z,t)d(z)\right]^{1/(1-\sigma)}$$

Price of the composite tradable good (P_{TR}):

$$\begin{aligned} P_{TR}^{1-\nu} &= \left(\frac{(1+\tau)w_F}{\kappa_{H,F}}\right)^{1-\nu}\int_{\mathbb{R}_+} z^{\nu-1} f_F(z,t)F\left(\frac{w\kappa_{H,F}}{(1+\tau)w_F}z,t\right)dz \\ &+w^{1-\nu}\int_{\mathbb{R}_+} z^{\nu-1} f(z,t)F_F\left(\frac{(1+\tau)w_F}{w\kappa_F}z,t\right)dz \end{aligned}$$

Home per capita consumption of the tradable and non-tradable composite is given by:

$$\begin{aligned} C_{TR} &= (1-\omega)\left(\frac{P_{TR}}{P}\right)^{-\eta}C \\ C_{NT} &= \omega\left(\frac{P_{NT}}{P}\right)^{-\eta}C \end{aligned}$$

Excess labor demand in the home country (E_H):

$$\begin{aligned} E_H(w,w_F) &= L_F w_F(1-\omega)\left(\frac{P_{TR}^F}{P^F}\right)^{1-\eta} D_{F,H} \\ &+L_H w\left[(1-\omega)\left(\frac{P_{TR}}{P}\right)^{1-\eta} D_{H,H} + \omega\left(\frac{P_{NT}}{P}\right)^{1-\eta}\right] - L_H w \end{aligned}$$

where The market share home country firms in the foreign country's tradable sector ($D_{H,F}$) and home country's tradable sector ($D_{H,H}$) is

$$\begin{aligned} D_{H,H} &= \int_0^\infty F_{TR}^F\left(\frac{w_F}{\kappa_{H,F}p}\right)d\left[1-F_{TR}\left(\frac{w}{p}\right)\right] \\ D_{F,H} &= \int_0^\infty F_{TR}^F\left(\frac{w_F}{p}\right)d\left[1-F_{TR}\left(\frac{w}{\kappa_{F,H}p}\right)\right] \end{aligned}$$

The foreign country static trade model is unchanged.

C.2 Growth model

The distribution of sellers in the home country's tradable sector is given by

$$G(z,t) = \int_0^z f_{TR}(y,t) F_{TR}^F\left(\frac{(1+\tau)w_F}{w_H \kappa_{H,F}} y\right) dy + \int_0^z f_{TR}^F(y,t) F_{TR}\left(\frac{w_H \kappa_{H,F}}{(1+\tau)w_F} y\right) dy$$

The differential equations for the home country productivity distributions are

$$\begin{aligned}
\frac{\partial \log[F_{NT}(z,t)]}{\partial t} &= \beta \log[G(z,t)] + \rho \log[F_{NT}(z,t)] \\
\frac{\partial \log[F_{TR}(z,t)]}{\partial t} &= \gamma \log[G(z,t)] + \delta \log[F_{NT}(z,t)]
\end{aligned}$$

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